

Code No: S3101

TOUGH · MODEL PAPER**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD****B. Tech III Year I Semester Examinations — Practice Model Paper 3 (TOUGH)****PROBABILITY AND STATISTICS**

(Common to CSE, IT) · Pattern: R18, as in the Apr/May-2026 paper

Time: 3 hours**Max. Marks: 75****Note:** i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.

iii) In Part B, answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A**(25 Marks)**

- 1.a)** If A and B are independent events, prove that A and $\bar{B}$ are also independent. **[2]**
- b)** Find the probability that a leap year selected at random has 53 Sundays or 53 Mondays. **[3]**
- c)** If $E(X) = 2$ and $Var(X) = 1$, find $E(3X - 4)$ and $Var(3X - 4)$. **[2]**
- d)** In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and s.d. **[3]**
- e)** Distinguish between point estimation and interval estimation. **[2]**
- f)** 40 engineers take a mean of 12.73 min with s.d. 2.06 min to assemble a part. With what confidence can we say that the sample mean differs from the true mean by at most 30 seconds? **[3]**
- g)** Why is the larger variance placed in the numerator of the F-statistic? **[2]**
- h)** S_1^2, S_2^2 are the variances of independent samples of sizes 10 and 15 from normal populations with equal variances. Find $P(S_1^2/S_2^2 < 4.03)$. **[3]**
- i)** Write the normal equations to fit $y = ab^x$. **[2]**
- j)** If $\sigma_y = 2\sigma_x$ and $r = 0.5$, find the tangent of the angle between the two regression lines. **[3]**

PART – B**(50 Marks)**

- 2.a)** A slip is given to X, who writes a plus sign with probability $\frac{1}{2}$, else a minus sign. X passes it to Y, Y to Z, Z to a referee. Each of Y and Z changes the sign with probability $\frac{1}{3}$. The referee sees a plus sign. Find the probability that X wrote a plus. **[6+4]**
- b)** A, B, C in that order toss a coin. The first to get a head wins. Find the chance of winning of each. **[6+4]**

OR

- 3.a)** A man speaks the truth 3 times out of 4. He throws a die and reports that it is a six. Find the probability that it is actually a six.
- b)** Urn A has 3 white and 2 black balls; urn B has 2 white and 4 black. One ball is transferred from A to B. Then a ball drawn from B is white. Find the probability that the transferred ball was white. **[5+5]**
- 4.a)** A continuous random variable has PDF $f(x) = kxe^{-\lambda x}$, $x \geq 0$, $\lambda > 0$. Find k, the mean, the variance and the mode.

- b)** 2% of the items made by a machine are defective. Using the Poisson distribution, find the probability that a box of 100 items has 3 or more defectives. **[6+4]**

OR

- 5.** A population consists of 2, 3, 6, 8, 11. List all samples of size 2 drawn **without replacement**. **[10]**
Find the mean and s.d. of the sampling distribution of means, and verify both using the formulas with the finite population correction.

- 6.a)** A maker claims that the mean breaking strength of his cables is at least 1800 kg. A sample of 50 cables gives mean 1750 kg with s.d. 120 kg. Test the claim at 1% level.

- b)** How large a sample is needed to be 99% confident that the error in estimating the mean is less than 20 kg ($\sigma = 120$)? **[6+4]**

OR

- 7.** In a sample of 400 people from town A, 120 smoke. In a sample of 600 from town B, 240 smoke. (a) Test at 5% whether the proportions of smokers differ. (b) Find the 95% confidence interval for the difference of the proportions. **[10]**

- 8.** Sample A: 66, 67, 75, 76, 82, 84, 88, 90, 92. Sample B: 64, 66, 74, 78, 82, 85, 87, 92, 93, 95, 97. Test whether the two samples come from the same normal population (use the F-test, then the t-test) at 5% level. **[10]**

OR

- 9.** Fit a Poisson distribution to the data and test the goodness of fit at 5%: x: 0, 1, 2, 3, 4; f: 109, 65, 22, 3, 1. (Pool classes where $E < 5$.) **[10]**

- 10.** Fit a parabola $y = a + bx + cx^2$ to the data: x: 0, 1, 2, 3, 4, 5, 6; y: 2.4, 2.1, 3.2, 5.6, 9.3, 14.6, 21.9. (Hint: put $X = x - 3$.) **[10]**

OR

- 11.a)** Compute Spearman's rank correlation: x: 48, 33, 40, 9, 16, 16, 65, 24, 16, 57; y: 13, 13, 24, 6, 15, 4, 20, 9, 6, 19.

- b)** The regression lines are $3x + 2y = 26$ and $6x + y = 31$. Find the means, identify the lines, and find r. **[6+4]**

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Answer Key

Part A

1a	$P(A \cap \overline{B}) = P(A) - P(A)P(B) = P(A)P(\overline{B})$.
1b	53 Sun: {Sat-Sun, Sun-Mon}; 53 Mon: {Sun-Mon, Mon-Tue}. Union has 3 of 7: $\frac{3}{7}$.
1c	$E = 3(2) - 4 = 2$; $Var = 9(1) = 9$.
1d	$z_1 = -0.5$, $z_2 = 1.405$; $\mu = 50$, $\sigma = 10$.
1e	See Unit 3, §3.2–3.3.
1f	S.E. = 0.3257; $z = 1.535$; confidence = $2(0.4376) = 87.5\%$.
1g	F-tables give only upper-tail values (> 1). See Unit 4, §4.6.
1h	$\nu = (9, 14)$; $F_{0.01}(9, 14) = 4.03$; probability = 0.99.
1i	$\log y = \log a + x \log b$; $nA + B\Sigma x = \Sigma Y$; $A\Sigma x + B\Sigma x^2 = \Sigma xY$; $a = 10^A$, $b = 10^B$.
1j	$\tan \theta = \frac{1-0.25}{0.5} \cdot \frac{2\sigma_y^2}{5\sigma_x^2} = 1.5 \times 0.4 = 0.6$.

Part B

Q2. Slip / cyclic

(a) Sign unchanged if 0 or 2 changes: $\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{5}{9}$. $P = \frac{\frac{1}{2} \cdot \frac{5}{9}}{\frac{1}{2} \cdot \frac{5}{9} + \frac{1}{2} \cdot \frac{1}{9}} = \frac{5}{9}$. (b) $\frac{4}{7}$, $\frac{2}{7}$, $\frac{1}{7}$.

Q3. Bayes

(a) $\frac{\frac{1}{6} \cdot \frac{3}{4}}{\frac{1}{6} \cdot \frac{3}{4} + \frac{5}{6} \cdot \frac{1}{4}} = \frac{3}{8}$. (b) $P(W) = \frac{3}{5} \cdot \frac{3}{7} + \frac{2}{5} \cdot \frac{2}{7} = \frac{13}{35}$; $P(\text{transferred W/W}) = \frac{9/35}{13/35} = \frac{9}{13}$.

Q4. PDF / Poisson

(a) $k = \lambda^2$; mean $\frac{2}{\lambda}$; variance $\frac{2}{\lambda}$; mode: $f'(x) = ke^{-\lambda x}(1 - \lambda x) = 0 \Rightarrow x = \frac{1}{\lambda}$. (b) $\lambda = 2$: $1 - e^{-2}(1 + 2 + 2) = 0.3233$.

Q5. Sampling w/o replacement

10 means: 2.5, 4, 5, 6.5, 4.5, 5.5, 7, 7, 8.5, 9.5. $\mu_{\bar{x}} = 6 = \mu$. $\sigma_{\bar{x}} = 2.012$. Formula: $\frac{3.286}{\sqrt{2}} \sqrt{\frac{3}{4}} = 2.012 \checkmark$

Q6. Left-tailed / sample size

(a) $H_1: \mu < 1800$. $Z = \frac{-50}{120/\sqrt{50}} = -2.95 < -2.33$: reject; claim not true. (b) $n = \left(\frac{2.58 \times 120}{20}\right)^2 = 239.6 \Rightarrow 240$.

Q7. Two proportions + CI

$p_1 = 0.30$, $p_2 = 0.40$, $P = 0.36$, S.E. = 0.0310, $Z = -3.23$: reject. CI uses $\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} = 0.0304$: $-0.10 \pm 0.0596 = (-0.160, -0.040)$.

Q8. F then t

$\bar{x} = 80, \bar{y} = 83$; SS 734, 1298; $S_1^2 = 91.75, S_2^2 = 129.8$. $F = 1.41 < F_{0.05}(10, 8) = 3.35$ ✓. $S^2 = \frac{2032}{18} = 112.9$; $t = -0.63, |t| < 2.101$ ✓. Same normal population.

Q9. Poisson fit

$\lambda = 0.61$. E: 108.67, 66.29, 20.22, 4.11, 0.71. Pool $x \geq 2$: O = 26, E = 25.04. $\chi^2 = 0.063$; $v = 3 - 1 - 1 = 1$; $0.063 < 3.841$: good fit.

Q10. Parabola

With $X = x - 3$: $\Sigma X = \Sigma X^3 = 0, \Sigma X^2 = 28, \Sigma X^4 = 196, \Sigma y = 59.1, \Sigma Xy = 89.6, \Sigma X^2y = 298$. $b = \frac{89.6}{28} = 3.2$; $7a + 28c = 59.1, 28a + 196c = 298 \rightarrow c = 0.733, a = 5.51$. Back to x : **$y = 2.51 - 1.2x + 0.733x^2$** .

Q11. Spearman / regression

(a) $\Sigma D^2 = 41$; ties: x has 16 ($\times 3$) $\rightarrow 2$; y has 13 ($\times 2$), 6 ($\times 2$) $\rightarrow 1$. $\rho = 1 - \frac{6(41+3)}{990} = 0.733$. (b) Means (4, 7). $3x + 2y = 26$ as y on x : $b_{yx} = -1.5$; $6x + y = 31$ as x on y : $b_{xy} = -\frac{1}{6}$. $r^2 = 0.25, r = -0.5$.