

Unit 1 — Probability

Sample space · Events · Axioms · Elementary theorems · Conditional probability · Bayes' theorem

Exam slot: Part-B Q2 / Q3 (10 M)

Part-A: 1a, 1b (2 + 3 M)

Written in ASD-STE100 style

PYQ TREND (2020 – 2026): WHAT THE EXAM ASKS FROM THIS UNIT

- **Bayes' theorem problem** (machines / hotels / boxes / VC): asked in **almost every paper**. Learn the table method.
- **Coloured balls in 3 boxes** (total probability): Apr-2026, May-2025.
- **Alternate tossing** (A and B toss in turn, who wins first): Apr-2026, Nov-2024, Oct-2023, Apr-2023, Jul-2021.
- **Two aircraft bomb a target** (0.3, 0.2): May-2025, Mar-2022.
- **Short questions**: axioms, sample space, conditional probability, Bayes statement, "spade or ace".

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|--|---|
| 1.1 Basic terms | 1.7 Multiplication theorem & independent events |
| 1.2 Types of events | 1.8 Total probability theorem |
| 1.3 Classical definition of probability | 1.9 Bayes' theorem |
| 1.4 Axioms of probability | 1.10 Quick revision sheet |
| 1.5 Elementary theorems (addition theorem) | ★ Assignment (5 questions) |
| 1.6 Conditional probability | |

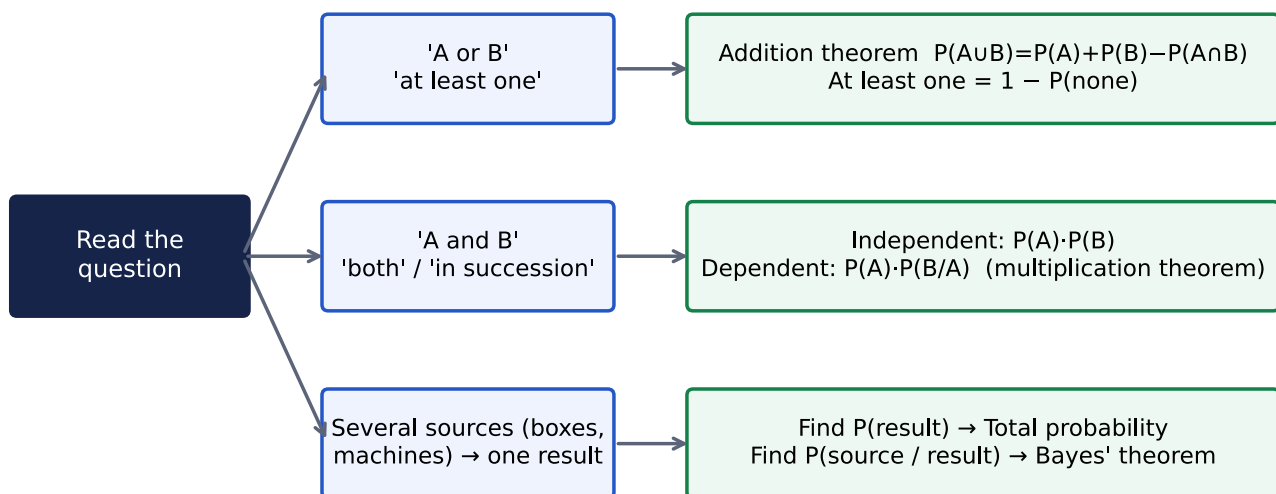


Fig 1.0 — Use this map to select the correct rule for a problem.

1.1 Basic Terms

RANDOM EXPERIMENT (RANDOM TRIAL)

An experiment that we do under the same conditions. We know the set of all possible outcomes. But the result is not certain before the experiment. The result is any one of the possible outcomes.

Example: Toss a coin. The result is H or T. We cannot tell which result will occur.

OUTCOME · ELEMENTARY EVENT · SIMPLE EVENT

Each result of a random experiment is an **outcome** (also called an **elementary event**). An event that we cannot split further is a **simple event**.

SAMPLE SPACE (S) AND SAMPLE POINT

The set of all possible simple events of a trial is the **sample space**. We write it as S . Each element of S is a **sample point**.

EVENT (E)

Any subset of the sample space is an **event**. We write it as E .

Example 1. Two coins are tossed. Write the sample space. Write the event "at least one head".

Class notes

$S = \{TT, TH, HT, HH\}$, so $n(S) = 4$.

$E = \text{at least one head} = \{TH, HT, HH\}$, so $n(E) = 3$.

PYQ. Define sample space and give an example. Oct-2025 · 2M Nov-2022 · 2M

Sample space: The set of all possible outcomes of a random experiment is the sample space S .

Example: Throw one die. $S = \{1, 2, 3, 4, 5, 6\}$. The event "even number" is $E = \{2, 4, 6\}$.

1.2 Types of Events

Equally likely events

No event has more chance to occur than the other events.

Example: A card from a pack: all 52 cards are equally likely. A coin: H and T are equally likely.

Exhaustive events

The set of all possible events of a trial.

Example: Coin: 2 exhaustive events (H, T). Die: 6 exhaustive events (1 to 6).

Mutually exclusive (disjoint) events

Two events E_1, E_2 cannot occur together: $E_1 \cap E_2 = \phi$.

Example: Two coins. $E_1 = \text{at least one head} = \{TH, HT, HH\}$, $E_2 = \text{exactly two tails} = \{TT\}$.
 $E_1 \cap E_2 = \phi$.

Complementary events

$E_1 \cap E_2 = \phi$ and $E_1 \cup E_2 = S$. The complement of E is $\bar{E}$.

If $n(S) = n$ and $n(E) = m$, then $n(\bar{E}) = n - m$.

Certain event

$P(E) = 1$. Example: Roll a die. $E = \text{number} \leq 6$.

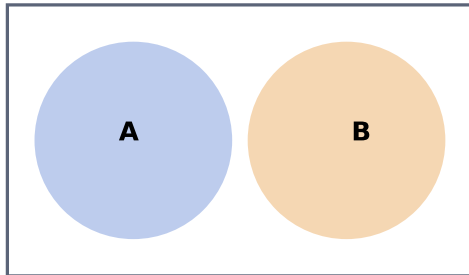
$$P(E) = 6/6 = 1.$$

Impossible event

$P(E) = 0$. Example: Roll a die. $E = \text{number} > 6$.

$$P(E) = 0.$$

Mutually exclusive: $A \cap B = \phi$



Not mutually exclusive: $A \cap B \neq \phi$

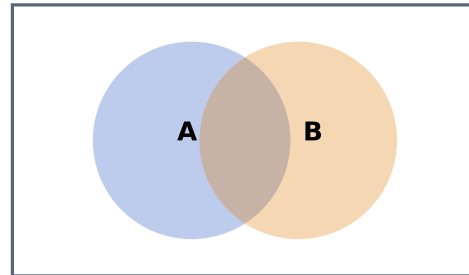


Fig 1.1 — Mutually exclusive events do not overlap.

1.3 Classical Definition of Probability

DEFINITION

A random experiment has n mutually exclusive, equally likely and exhaustive outcomes. Of these, m outcomes are favourable to the event E . Then the probability of E is:

$$P(E) = \frac{\text{No. of favourable outcomes to } E}{\text{Total no. of outcomes}} = \frac{n(E)}{n(S)} = \frac{m}{n}$$

RANGE OF PROBABILITY AND COMPLEMENT RULE

Because $0 \leq m \leq n$, divide by n : $0 \leq \frac{m}{n} \leq 1$. So $0 \leq P(E) \leq 1$.

$$P(\bar{E}) = \frac{n-m}{n} = 1 - \frac{m}{n}. \text{ So } \boxed{P(E) + P(\bar{E}) = 1}.$$

NOTE — "AT LEAST ONE" PROBLEMS

For "at least one", use the complement. $P(\text{at least one}) = 1 - P(\text{none})$. This method is short and safe.

Example 2. Find the probability that a leap year has 53 Sundays. [Class notes](#)

A leap year has 366 days = 52 weeks + 2 days. The 2 extra days can be:

(Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun). So $n(S) = 7$.

$E = 53 \text{ Sundays} = \{(\text{Sun, Mon}), (\text{Sat, Sun})\}$. So $n(E) = 2$.

$$P(E) = \frac{2}{7}. \quad \boxed{2/7}$$

(For "52 Mondays and 53 Sundays", only (Sat, Sun) is favourable, so $P = 1/7$.)

Example 3. In a class there are 10 boys and 5 girls. We select a committee of 4 students. Find the probability that the committee has (i) at least 3 girls (ii) exactly 3 girls (iii) at most 3 girls (iv) 2 or 3 girls. [Class notes](#)

Total students = 15. $n(S) = {}^{15}C_4 = 1365$.

$$1. \text{ At least 3 girls: } n(E) = {}^5C_3 \cdot {}^{10}C_1 + {}^5C_4 \cdot {}^{10}C_0 = 100 + 5 = 105. \quad P = \frac{105}{1365} = \frac{1}{13}.$$

2. Exactly 3 girls: $n(E) = {}^5C_3 \cdot {}^{10}C_1 = 100$. $P = \frac{100}{1365} = \frac{20}{273}$.
3. At most 3 girls = $1 - P(4 \text{ girls}) = 1 - \frac{{}^5C_4}{{}^{13}C_4} = 1 - \frac{5}{1365} = \frac{1360}{1365} = \frac{272}{273}$.
4. 2 or 3 girls: $n(E) = {}^5C_2 \cdot {}^{10}C_2 + {}^5C_3 \cdot {}^{10}C_1 = 450 + 100 = 550$. $P = \frac{550}{1365} = \frac{110}{273}$.

PYQ. A five-figure number is formed by the digits 0, 1, 2, 3, 4 (without repetition). Find the probability that the number is divisible by 4. **Nov-2024 · 6M**

Step 1 Find $n(S)$. The first place cannot be 0. So $n(S) = 4 \times 4 \times 3 \times 2 \times 1 = 96$.

Step 2 A number is divisible by 4 when its **last two digits** are divisible by 4. Possible endings: 04, 12, 20, 24, 32, 40.

Step 3 Endings with 0 (04, 20, 40): the other 3 digits fill 3 places freely: $3! = 6$ each. Total = 18.

Step 4 Endings without 0 (12, 24, 32): 0 is left. First place cannot be 0: $2 \times 2 \times 1 = 4$ each. Total = 12.

Step 5 $n(E) = 18 + 12 = 30$. $P(E) = \frac{30}{96} = \frac{5}{16}$. **5/16**

PYQ. What is the probability of getting 9 cards of the same suit if 12 cards are drawn at random?

Mar/Apr-2024 · 3M

$n(S) = {}^{52}C_{12}$. Select the suit in 4 ways. Select 9 cards from that suit: ${}^{13}C_9$. Select 3 other cards from the other 39 cards: ${}^{39}C_3$.

$$P = \frac{4 \cdot {}^{13}C_9 \cdot {}^{39}C_3}{{}^{52}C_{12}} \approx 0.000127$$

PYQ. A die is rolled. If the number is odd, what is the probability that it is prime? **Nov-2024 · 3M**

The number is odd, so the new sample space is $\{1, 3, 5\}$. $n(S) = 3$. Odd primes are $\{3, 5\}$. $n(E) = 2$.

P = 2/3

PYQ. In a sample of 446 cars stopped at a road block, only 67 drivers had their seat belts fastened. Estimate the probability that a driver will have the seat belt fastened. **Apr/May-2026 · 3M**

$n(S) = 446$, $n(E) = 67$. $P(E) = \frac{67}{446} \approx 0.150$. **≈ 0.15**

PYQ. (a) Ten coins are tossed. Find the probability of at least eight heads. (b) A coin is tossed 12 times. Find the probability of at least one head. **Apr/May-2023 · 2M** **Nov-2022 · 2M**

(a) $n(S) = 2^{10} = 1024$. $n(E) = {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} = 45 + 10 + 1 = 56$. $P = \frac{56}{1024} = \frac{7}{128}$.

(b) $P = 1 - P(\text{no head}) = 1 - \frac{1}{2^{12}} = \frac{4095}{4096}$.

1.4 Axioms of Probability

DEFINITION (AXIOMATIC APPROACH)

Let S be a finite sample space. A real-valued function P from the power set of S into $\mathbb{R}$, $P : \mathcal{P}(S) \rightarrow \mathbb{R}$, is a **probability function** on S if it satisfies these 3 axioms:

Axiom	Statement	Meaning
1. Positivity	$P(E) \geq 0$ for every event E in S	A probability is never negative.
2. Certainty	$P(S) = 1$	The sample space always occurs.
3. Union (additivity)	If $E_1 \cap E_2 = \phi$, then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$	For disjoint events, add the probabilities.

NOTE — EXTENDED AXIOM OF UNION

If $E_1, E_2, \dots, E_n$ are pairwise disjoint ($E_i \cap E_j = \phi$ for all $i \neq j$), then $P(\bigcup_{i=1}^n E_i) = \sum_{i=1}^n P(E_i)$.

PYQ. State and prove any two axioms of probability. Oct-2025 · 5M Apr/May-2026 · 2M (state)

Axiom 2 (Certainty): $P(S) = 1$.

Proof: $n(S)$ outcomes are all favourable to S . So $P(S) = \frac{n(S)}{n(S)} = 1$.

Axiom 3 (Union): If $E_1 \cap E_2 = \phi$, then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$.

Proof: $P(E_1 \cup E_2) = \frac{n(E_1 \cup E_2)}{n(S)}$.

By the principle of inclusion and exclusion, $n(E_1 \cup E_2) = n(E_1) + n(E_2) - n(E_1 \cap E_2) = n(E_1) + n(E_2) - 0$.

So $P(E_1 \cup E_2) = \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)} = P(E_1) + P(E_2)$. ■

Axiom 1 (Positivity): $P(E) \geq 0$. Proof: $n(E) \geq 0$ and $n(S) > 0$. So $P(E) = n(E)/n(S) \geq 0$.

Example 4. A card is drawn from a pack of 52 cards. Find the probability that it is a queen or a king.

[Class notes](#)

$n(S) = 52$. E_1 = queen: $P(E_1) = 4/52 = 1/13$. E_2 = king: $P(E_2) = 1/13$.

A card cannot be a queen and a king. So $E_1 \cap E_2 = \phi$. By the axiom of union:

$$P(E_1 \cup E_2) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}. \quad \text{2/13}$$

Example 5. Students A, B, C run a race. A and B have the same chance to win. Each is twice as likely to win as C. Find the probability that B or C wins. [Class notes](#)

$P(A) = P(B) = 2P(C)$. Only one student wins, so A, B, C are mutually exclusive and $A \cup B \cup C = S$.

$$P(A) + P(B) + P(C) = 1 \Rightarrow 2P(C) + 2P(C) + P(C) = 1 \Rightarrow P(C) = \frac{1}{5}, P(A) = P(B) = \frac{2}{5}.$$

$$P(B \cup C) = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}. \quad \text{3/5}$$

1.5 Some Elementary Theorems

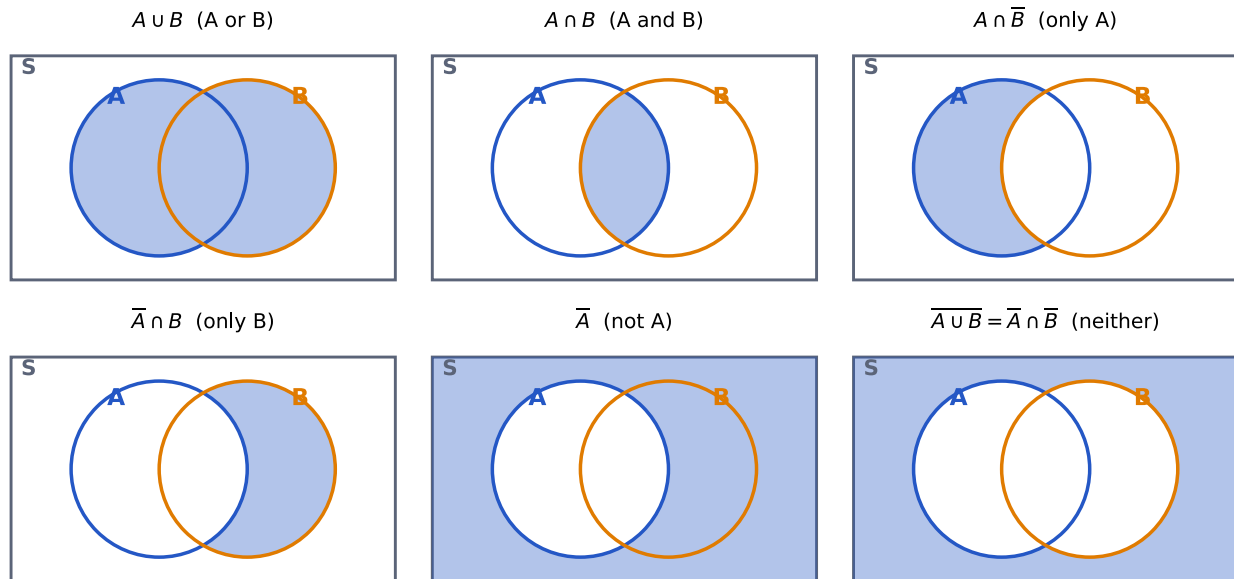


Fig 1.2 — Venn diagrams for the main event combinations (shaded part = event).

ELEMENTARY THEOREMS (LEARN THESE 6)

1. $P(\phi) = 0$ (impossible event).
2. $P(\bar{A}) = 1 - P(A)$.
3. If $A \subseteq B$, then $P(A) \leq P(B)$.
4. $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ and $P(\bar{A} \cap B) = P(B) - P(A \cap B)$.
5. **Addition theorem:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
6. **De Morgan:** $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$ and $P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$.

Addition theorem — statement and proof Nov-2022 · 3M

Statement: Let A and B be two events in a sample space S . Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Proof.

Case 1: $A \cap B = \phi$. Then $P(A \cap B) = 0$. By the axiom of union, $P(A \cup B) = P(A) + P(B) = P(A) + P(B) - 0$. The result is true.

Case 2: $A \cap B \neq \phi$. Write $A \cup B = A \cup (\bar{A} \cap B)$, where $A \cap (\bar{A} \cap B) = \phi$.

By the axiom of union: $P(A \cup B) = P(A) + P(\bar{A} \cap B)$.

Add and subtract $P(A \cap B)$: $P(A \cup B) = P(A) + [P(\bar{A} \cap B) + P(A \cap B)] - P(A \cap B)$.

The events $(\bar{A} \cap B)$ and $(A \cap B)$ are disjoint, and $(\bar{A} \cap B) \cup (A \cap B) = B$. So the bracket is $P(B)$.

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$. ■

ADDITION THEOREM FOR 3 EVENTS

$$\begin{aligned}
 P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\
 &\quad - P(A \cap B) - P(B \cap C) - P(A \cap C) \\
 &\quad + P(A \cap B \cap C)
 \end{aligned}$$

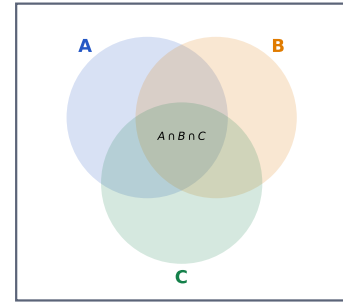


Fig 1.3 — Three events

PYQ. A card is drawn from a well-shuffled pack of cards. Find the probability that it is either a spade or an ace. May-2025 · 3M Oct-2023 · 3M

$$\begin{aligned}
 n(S) &= 52. E_1 = \text{spade: } P(E_1) = \frac{13}{52}. E_2 = \text{ace: } P(E_2) = \frac{4}{52}. E_1 \cap E_2 = \text{ace of spades: } P = \frac{1}{52}. \\
 P(E_1 \cup E_2) &= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}. \quad \text{4/13}
 \end{aligned}$$

PYQ. 60% of people like tea, 50% like coffee and 30% like both. Find the probability that a person likes at least one of the two. Oct-2025 · 5M

$$P(T \cup C) = P(T) + P(C) - P(T \cap C) = 0.6 + 0.5 - 0.3 = 0.8. \quad \text{0.8}$$

PYQ. Given $P(A) = 0.30$, $P(B) = 0.62$, $P(A \cap B) = 0.12$. Find (i) $P(A \cup B)$ (ii) $P(\bar{A} \cap B)$ (iii) $P(A \cap \bar{B})$ (iv) $P(\bar{A} \cup \bar{B})$. Nov/Dec-2022 · 5M

- (i) $P(A \cup B) = 0.30 + 0.62 - 0.12 = 0.80$
- (ii) $P(\bar{A} \cap B) = P(B) - P(A \cap B) = 0.62 - 0.12 = 0.50$
- (iii) $P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.30 - 0.12 = 0.18$
- (iv) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - 0.12 = 0.88$

PYQ. If $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{6}$, find (i) $P(A \cup B)$ (ii) $P(A \cap B^c)$ (iii) $P(A \cap B)^c$.

Jul-2021 · 7M

$$(i) \frac{1}{4} + \frac{2}{3} - \frac{1}{6} = \frac{3+8-2}{12} = \frac{3}{4} \quad (ii) \frac{1}{4} - \frac{1}{6} = \frac{1}{12} \quad (iii) 1 - \frac{1}{6} = \frac{5}{6}$$

Example 6. In a city, newspapers A, B, C are read by 20%, 16%, 14%. A and B: 8%, A and C: 5%, B and C: 4%, all three: 2%. Find the % of people who read at least one paper. Class notes

$$P(A \cup B \cup C) = 0.20 + 0.16 + 0.14 - 0.08 - 0.05 - 0.04 + 0.02 = 0.35. \quad \text{35\%}$$

PYQ. Show that for any two events A and B, $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$.

Apr/May-2023 · 5M

Step 1 $A \cap B \subseteq A$. By theorem 3, $P(A \cap B) \leq P(A)$.

Step 2 $A \subseteq A \cup B$. By theorem 3, $P(A) \leq P(A \cup B)$.

Step 3 By the addition theorem, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Because $P(A \cap B) \geq 0$ (axiom 1), $P(A \cup B) \leq P(A) + P(B)$.

Step 4 Join the three results: $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$. ■

1.6 Conditional Probability

CONDITIONAL EVENT AND CONDITIONAL PROBABILITY

Let E_1, E_2 be two events in S with $P(E_1) \neq 0$. The event " E_2 occurs after E_1 has occurred" is a **conditional event**. We write it as E_2/E_1 .

The probability of E_2 , given that E_1 has occurred, is the **conditional probability**:

$$P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{n(E_1 \cap E_2)}{n(E_1)}$$

Textbooks also write $P(E_2 | E_1)$. It has the same meaning.

NOTE — HOW TO READ IT

"Given E_1 " means: E_1 becomes the **new sample space**. Count only inside E_1 .

Examples of conditional events: Two dice are thrown and the sum is 7; find the chance of a 1 on one die. Two coins are tossed; find the chance of 2 tails, given at least one tail.

Two dice: $n(S) = 36$
blue: sum 6 (5 ways) orange: sum 7 (6 ways)

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Fig 1.4 — Sample space of two dice. Use it for sum, maximum and minimum problems.

Example 7. A die is rolled. $A = \{1, 3, 5\}$, $B = \{2, 3\}$, $C = \{2, 3, 4, 5\}$. Find $P(A/B)$, $P(B/A)$, $P(A/C)$, $P(C/A)$, $P(B/C)$, $P(C/B)$. [Class notes](#)

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{2}{3}$. $A \cap B = \{3\}$, $A \cap C = \{3, 5\}$, $B \cap C = \{2, 3\}$.

$P(A/B)$	$P(B/A)$	$P(A/C)$	$P(C/A)$	$P(B/C)$	$P(C/B)$
$\frac{1/6}{1/3} = \frac{1}{2}$	$\frac{1/6}{1/2} = \frac{1}{3}$	$\frac{1/3}{2/3} = \frac{1}{2}$	$\frac{1/3}{1/2} = \frac{2}{3}$	$\frac{1/3}{2/3} = \frac{1}{2}$	$\frac{1/3}{1/3} = 1$

Example 8. If $P(\bar{A}) = \frac{3}{8}$, $P(\bar{B}) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{4}$, find (i) $P(A/B)$, $P(B/A)$ (ii) $P(\bar{A}/\bar{B})$, $P(\bar{B}/\bar{A})$.

Class notes (H.W.)

$$P(A) = \frac{5}{8}, P(B) = \frac{1}{2}.$$

$$(i) P(A/B) = \frac{1/4}{1/2} = \frac{1}{2}; P(B/A) = \frac{1/4}{5/8} = \frac{2}{5}.$$

$$(ii) P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - \left[\frac{5}{8} + \frac{1}{2} - \frac{1}{4} \right] = 1 - \frac{7}{8} = \frac{1}{8}.$$

$$P(\bar{A}/\bar{B}) = \frac{1/8}{1/2} = \frac{1}{4}; P(\bar{B}/\bar{A}) = \frac{1/8}{3/8} = \frac{1}{3}.$$

PYQ. Define conditional probability.

Nov-2024 · 2M

May-2025 · 3M

Let A and B be two events with $P(A) \neq 0$. The probability of B , given that A has occurred, is the conditional probability of B given A : $P(B/A) = \frac{P(A \cap B)}{P(A)}$.

1.7 Multiplication Theorem & Independent Events

MULTIPLICATION THEOREM

Let E_1, E_2 be events in S with $P(E_1) \neq 0$, $P(E_2) \neq 0$. Then

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1) \quad \text{or} \quad P(E_1 \cap E_2) = P(E_2) \cdot P(E_1/E_2)$$

Proof: By the definition of conditional probability, $P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$. Multiply both sides by $P(E_1)$. ■

INDEPENDENT AND DEPENDENT EVENTS

E_2 is **independent** of E_1 if the occurrence of E_1 does not change the probability of E_2 : $P(E_2/E_1) = P(E_2)$.

If E_2 is independent of E_1 , then E_1 is also independent of E_2 . We say E_1, E_2 are **(mutually) independent**.

E_2 is **dependent** on E_1 if $P(E_2/E_1) \neq P(E_2)$.

Compound event: two or more events occur together (example: "4 on a die and head on a coin").

CAUTION — INDEPENDENT ≠ MUTUALLY EXCLUSIVE

Independent: $P(A \cap B) = P(A)P(B)$. Both events can occur together.

Mutually exclusive: $P(A \cap B) = 0$. Both events cannot occur together.

For 3 events, $P(A \cap B \cap C) = P(A)P(B)P(C)$ needs **mutual** independence. Pairwise independence alone is not sufficient.

Theorem 1: If A, B are independent, then $P(A \cap B) = P(A) \cdot P(B)$

Oct-2025 · 5M

A, B are independent, so by definition $P(B/A) = P(B) \dots (1)$.

By the multiplication theorem, $P(A \cap B) = P(A) \cdot P(B/A)$. Use (1): $P(A \cap B) = P(A) \cdot P(B)$. ■

Theorem 2: If A, B are independent, then $\bar{A}, \bar{B}; \bar{A}, B; A, \bar{B}$ are also independent

Apr/May-2023 · 3M

Given $P(A \cap B) = P(A)P(B) \dots (1)$.

- (i) $\bar{A}, \bar{B}$: $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A)P(B)]$
 $= [1 - P(A)] - P(B)[1 - P(A)] = [1 - P(A)][1 - P(B)] = P(\bar{A})P(\bar{B})$. So $\bar{A}, \bar{B}$ are independent.
- (ii) $\bar{A}, B$: $P(\bar{A} \cap B) = P(B) - P(A \cap B) = P(B) - P(A)P(B) = P(B)[1 - P(A)] = P(\bar{A})P(B)$.
- (iii) $A, \bar{B}$: $P(A \cap \bar{B}) = P(A) - P(A \cap B) = P(A) - P(A)P(B) = P(A)[1 - P(B)] = P(A)P(\bar{B})$. ■

PYQ. Find the probability of drawing two red balls in succession from a bag of 4 red and 5 black balls, (i) with replacement (ii) without replacement. Apr/May-2026 · 4M Class notes

Total balls = 9. E_1 = first ball red, E_2 = second ball red.

(i) **With replacement:** $P(E_1) = \frac{4}{9}$, $P(E_2/E_1) = \frac{4}{9}$. $P(E_1 \cap E_2) = \frac{4}{9} \cdot \frac{4}{9} = \frac{16}{81}$.

(ii) **Without replacement:** $P(E_1) = \frac{4}{9}$. After one red ball goes out, 3 red of 8 remain: $P(E_2/E_1) = \frac{3}{8}$.

By the multiplication theorem, $P(E_1 \cap E_2) = \frac{4}{9} \cdot \frac{3}{8} = \frac{1}{6}$. (i) 16/81 (ii) 1/6

PYQ. A box has 4 red and 6 blue balls. Two balls are drawn without replacement. Find the probability that both are red. Oct-2025 · 5M

$$P = \frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15} \quad (\text{or } \frac{{}^4C_2}{{}^{10}C_2} = \frac{6}{45} = \frac{2}{15}). \quad \text{2/15}$$

PYQ. Two marbles are drawn in succession from a box of 10 red, 15 white, 20 blue and 30 orange marbles, without replacement. Find the probability that (i) both are orange (ii) neither is white.

Mar/Apr-2024 · 5M

Total = 75.

$$(i) P = \frac{30}{75} \cdot \frac{29}{74} = \frac{29}{185}. \quad (ii) \text{ Non-white} = 60: P = \frac{60}{75} \cdot \frac{59}{74} = \frac{118}{185}.$$

PYQ. Two aeroplanes bomb a target in succession. The probability that each scores a hit is 0.3 and 0.2. The second plane bombs only if the first misses. Find the probability that (i) the target is hit (ii) the target is hit by the second plane (iii) both fail. May-2025 · 7M Mar/Apr-2022 · 8M

$$P(A) = 0.3, P(\bar{A}) = 0.7, P(B) = 0.2, P(\bar{B}) = 0.8.$$

Step 1 Target is hit = "A hits" or "A misses and then B hits": $P[A \cup (\bar{A} \cap B)] = P(A) + P(\bar{A})P(B) = 0.3 + 0.7(0.2) = 0.44$.

Step 2 Hit by the second plane = $P(\bar{A} \cap B) = 0.7 \times 0.2 = 0.14$.

Step 3 Both fail = $P(\bar{A} \cap \bar{B}) = 0.7 \times 0.8 = 0.56$. (Check: $0.44 + 0.56 = 1$.)

(i) 0.44 (ii) 0.14 (iii) 0.56

PYQ. A, B and C aim to shoot a balloon. A succeeds 4 times in 5, B 3 times in 4, C 2 times in 3. If all three aim together, find the probability that at least two of them hit. Nov/Dec-2022 · 5M Class notes

$$P(A) = \frac{4}{5}, P(\bar{A}) = \frac{1}{5}; P(B) = \frac{3}{4}, P(\bar{B}) = \frac{1}{4}; P(C) = \frac{2}{3}, P(\bar{C}) = \frac{1}{3}.$$

At least two = exactly two + all three:

$$P = P(ABC) + P(AB\bar{C}) + P(A\bar{B}C) + P(\bar{A}BC)$$

$$= \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{1}{4} \cdot \frac{2}{3} + \frac{1}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} = \frac{2}{5} + \frac{1}{5} + \frac{2}{15} + \frac{1}{10} = \frac{5}{6}. \quad \text{5/6}$$

Example 9. The probabilities that students A, B, C, D solve a problem are $\frac{1}{3}, \frac{2}{5}, \frac{1}{5}, \frac{1}{4}$. All try. Find the probability that the problem is solved. [Class notes](#)

Solved = at least one solves. $P = 1 - P(\overline{A})P(\overline{B})P(\overline{C})P(\overline{D}) = 1 - \frac{2}{3} \cdot \frac{3}{5} \cdot \frac{4}{5} \cdot \frac{3}{4} = 1 - \frac{6}{25} = \frac{19}{25}$.

PYQ. Of 10 girls in a class, 3 have blue eyes. 2 girls are chosen at random. Find the probability that (i) both have blue eyes (ii) at least one has blue eyes. [Oct-2023 · 3M](#)

$n(S) = {}^{10}C_2 = 45$. (i) ${}^3C_2/45 = 3/45 = \frac{1}{15}$. (ii) $1 - {}^7C_2/45 = 1 - \frac{21}{45} = \frac{8}{15}$.

PYQ. Compute the probability that at least two 7's appear (as a sum) in four tosses of a pair of fair dice. [Mar/Apr-2024 · 5M](#)

From Fig 1.4, $P(\text{sum } 7) = \frac{6}{36} = \frac{1}{6} = p$, $q = \frac{5}{6}$. Each toss is independent.

$P(\text{at least two}) = 1 - P(0) - P(1) = 1 - \left(\frac{5}{6}\right)^4 - 4 \cdot \frac{1}{6} \left(\frac{5}{6}\right)^3 = 1 - \frac{625}{1296} - \frac{500}{1296} = \frac{171}{1296} = \frac{19}{144}$.

PYQ. Box A has 5 red and 3 white marbles. Box B has 2 red and 6 white marbles. One marble is drawn from each box. Find the probability that both are of the same colour. [Class notes](#)

The two draws are independent.

Both red: $\frac{5}{8} \cdot \frac{2}{8} = \frac{10}{64}$. Both white: $\frac{3}{8} \cdot \frac{6}{8} = \frac{18}{64}$.

Same colour = $\frac{10}{64} + \frac{18}{64} = \frac{28}{64} = \frac{7}{16}$. **7/16**

CAUTION

A marble is drawn from **each** box. We do not select a box. So do not multiply by $\frac{1}{2}$ here.

Alternate tossing problems (frequent 5–7 M PYQ)

A wins on toss 1, 3, 5, ... $\rightarrow P(\text{A wins}) = p + q^2p + q^4p + \dots = p / (1 - q^2)$

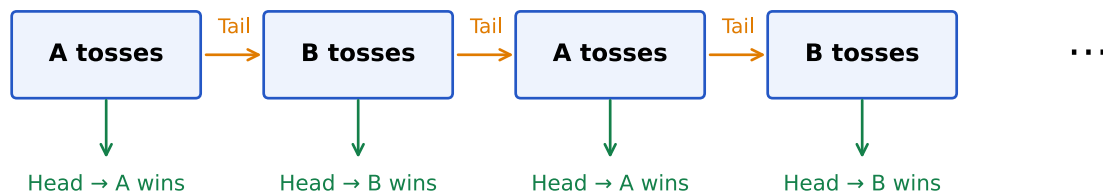


Fig 1.5 — A wins on turns 1, 3, 5, ... Each earlier turn must be a failure for both players.

FORMULA — GEOMETRIC SERIES

$1 + r + r^2 + r^3 + \dots = \frac{1}{1 - r}$ when $|r| < 1$.

If A succeeds with probability p_1 and B with p_2 ($q_1 = 1 - p_1$, $q_2 = 1 - p_2$), and A starts:

$$P(\text{A wins}) = p_1 + q_1q_2p_1 + (q_1q_2)^2p_1 + \dots = \frac{p_1}{1 - q_1q_2}$$

PYQ. A and B toss a fair coin alternately. The first to get a head wins. If A starts, find his chance of winning. Apr/May-2023 · 5M Jul-2021 · 8M

$$p_1 = p_2 = \frac{1}{2}, q_1 q_2 = \frac{1}{4}. P(A) = \frac{1/2}{1 - 1/4} = \frac{2}{3}. \text{ Then } P(B) = 1 - \frac{2}{3} = \frac{1}{3}. \quad \boxed{2/3}$$

PYQ. A man alternately tosses a coin and throws a die, beginning with the coin. Find the probability that he gets a head before he gets a 3 or 4 on the die. Apr/May-2026 · 6M Nov-2024 · 4M (5 or 6)

Coin: $p_1 = P(H) = \frac{1}{2}, q_1 = \frac{1}{2}$. Die: $p_2 = P(3 \text{ or } 4) = \frac{2}{6} = \frac{1}{3}, q_2 = \frac{2}{3}$.

He wins when the head comes first: on turn 1, 3, 5, ...

$$P = \frac{1}{2} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{2} + \left(\frac{1}{2} \cdot \frac{2}{3}\right)^2 \frac{1}{2} + \dots = \frac{1/2}{1 - \frac{1}{2} \cdot \frac{2}{3}} = \frac{1/2}{2/3} = \frac{3}{4}.$$

For "5 or 6" (Nov-2024), $p_2 = \frac{1}{3}$ again, so the answer is also $\frac{3}{4}$. 3/4

PYQ. A and B throw alternately with a pair of dice. A wins if he throws 6 before B throws 7. B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is 30/61. Oct-2023 · 7M

[Class notes](#)

From Fig 1.4: sum 6 = {(1,5),(5,1),(2,4),(4,2),(3,3)}: $p_1 = \frac{5}{36}, q_1 = \frac{31}{36}$.

Sum 7 has 6 ways: $p_2 = \frac{1}{6}, q_2 = \frac{5}{6}$.

$$P(A) = \frac{p_1}{1 - q_1 q_2} = \frac{5/36}{1 - \frac{31}{36} \cdot \frac{5}{6}} = \frac{5/36}{1 - \frac{155}{216}} = \frac{5}{36} \cdot \frac{216}{61} = \frac{30}{61}. \blacksquare$$

Example 10. A, B, C in order toss a coin. The first to toss a head wins. The game may continue for ever. Find each player's chance of winning. Class notes

One full round of failures has probability $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

$$P(A) = \frac{1}{2} \left[1 + \frac{1}{8} + \frac{1}{8^2} + \dots\right] = \frac{1}{2} \cdot \frac{1}{1-1/8} = \frac{4}{7}; \quad P(B) = \frac{1}{4} \cdot \frac{8}{7} = \frac{2}{7}; \quad P(C) = \frac{1}{8} \cdot \frac{8}{7} = \frac{1}{7}.$$

1.8 Theorem of Total Probability

STATEMENT

Let $E_1, E_2, \dots, E_n$ be mutually exclusive and exhaustive events with $P(E_i) > 0$. Let A be any event in S . Then

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \dots + P(E_n)P(A/E_n) = \sum_{i=1}^n P(E_i) P(A/E_i)$$

Tree diagram: Total probability = sum of all branch products ending in D

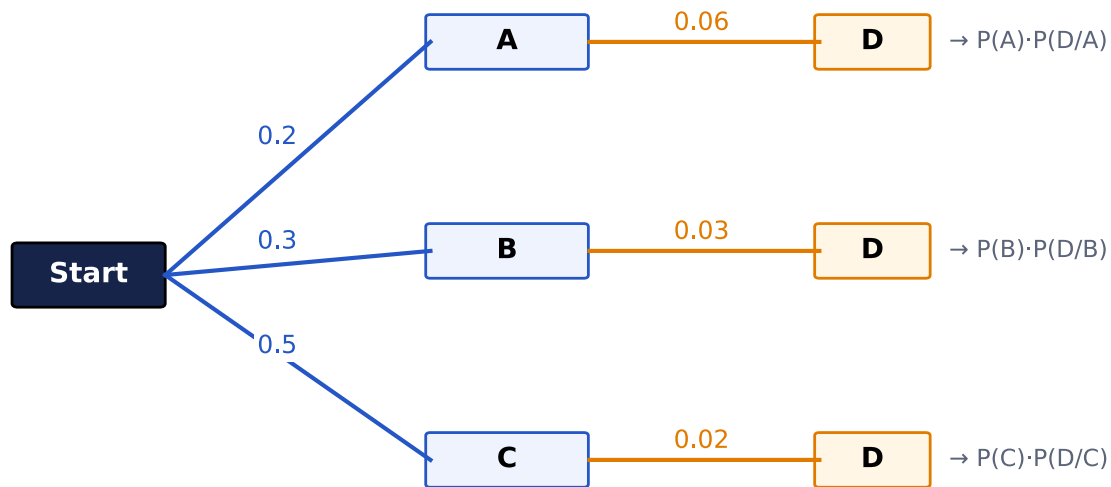


Fig 1.6 — Tree diagram. Each path = $P(\text{source}) \times P(\text{result} / \text{source})$. Add all paths for $P(\text{result})$.

PYQ. Coloured balls are in three boxes: Box 1 (4 white, 1 red, 3 blue), Box 2 (2 white, 3 red, 5 blue), Box 3 (3 white, 4 red, 3 blue). A box is selected at random and a ball is drawn. Find the probability that the ball is (a) red (b) blue. Apr/May-2026 · 10M Class notes

	Box X	Box Y	Box Z
White	4	2	3
Red	1	3	4
Blue	3	5	3
Total	8	10	10

Step 1 Each box has the same chance: $P(X) = P(Y) = P(Z) = \frac{1}{3}$.

Step 2 Let R = red ball. $P(R/X) = \frac{1}{8}$, $P(R/Y) = \frac{3}{10}$, $P(R/Z) = \frac{4}{10}$.

Step 3 By total probability: $P(R) = \frac{1}{3} \cdot \frac{1}{8} + \frac{1}{3} \cdot \frac{3}{10} + \frac{1}{3} \cdot \frac{4}{10} = \frac{1}{3} \left[\frac{5+12+16}{40} \right] = \frac{33}{120} = \frac{11}{40}$.

Step 4 Let B = blue. $P(B/X) = \frac{3}{8}$, $P(B/Y) = \frac{5}{10}$, $P(B/Z) = \frac{3}{10}$.
 $P(B) = \frac{1}{3} \left[\frac{3}{8} + \frac{5}{10} + \frac{3}{10} \right] = \frac{1}{3} \left[\frac{15+20+12}{40} \right] = \frac{47}{120}$.

(a) 11/40 (b) 47/120

PYQ. Box I: 2 red, 3 white, 5 blue. Box II: 4 red, 1 white, 3 blue. Box III: 3 red, 4 white, 3 blue. A box is selected at random and a ball is drawn. Find the probability that the ball is (a) red (b) blue.

May-2025 · 10M

Totals: 10, 8, 10. $P(R) = \frac{1}{3} \left[\frac{2}{10} + \frac{4}{8} + \frac{3}{10} \right] = \frac{1}{3} [0.2 + 0.5 + 0.3] = \frac{1}{3}$.

$P(B) = \frac{1}{3} \left[\frac{5}{10} + \frac{3}{8} + \frac{3}{10} \right] = \frac{1}{3} [0.5 + 0.375 + 0.3] = \frac{47}{120}$. **(a) 1/3 (b) 47/120**

Example 11. Machines I, II, III produce 40%, 30%, 30% of the items. Their defective rates are 4%, 2%, 3%. Find the probability that an item selected at random is defective. [Class notes](#)

$$P(D) = 0.4(0.04) + 0.3(0.02) + 0.3(0.03) = 0.016 + 0.006 + 0.009 = 0.031. \quad \mathbf{0.031 = 31/1000}$$

Example 12. Parties X, Y, Z have chances 1 : 2 : 3 to win. If selected, the probability of total prohibition is $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$. Find the probability of total prohibition. [Class notes](#)

$$P(X) = \frac{1}{6}, P(Y) = \frac{2}{6}, P(Z) = \frac{3}{6}. P(A) = \frac{1}{6} \cdot \frac{1}{2} + \frac{2}{6} \cdot \frac{1}{4} + \frac{3}{6} \cdot \frac{3}{4} = \frac{2+2+9}{24} = \frac{13}{24}.$$

1.9 Bayes' Theorem ★★ (10 M)

STATEMENT OCT-2025 · 3M

Let $E_1, E_2, \dots, E_n$ be n mutually exclusive and exhaustive events with $P(E_i) > 0, 1 \leq i \leq n$, in a sample space S . Let A be any other event in S that can occur only with one of the E_i , and $P(A) > 0$. If $P(E_i)$ and $P(A/E_i)$ are known, then for any E_k :

$$P(E_k/A) = \frac{P(E_k) P(A/E_k)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \dots + P(E_n)P(A/E_n)} = \frac{P(E_k) P(A/E_k)}{\sum_{i=1}^n P(E_i)P(A/E_i)}$$

Significance: Bayes' theorem gives the probability of a **cause** (E_k) after we see the **effect** (A). It updates a prior probability $P(E_k)$ into a posterior probability $P(E_k/A)$.

Proof

Nov/Dec-2022 · 10M
Nov-2020

- Step 1** $E_1, \dots, E_n$ are mutually exclusive and exhaustive. So $E_i \cap E_j = \phi$ for $i \neq j$, and $E_1 \cup E_2 \cup \dots \cup E_n = S$.
- Step 2** A is any event in S . So $A = A \cap S = A \cap (E_1 \cup E_2 \cup \dots \cup E_n) = (A \cap E_1) \cup (A \cap E_2) \cup \dots \cup (A \cap E_n)$.
- Step 3** $(A \cap E_i) \cap (A \cap E_j) = A \cap (E_i \cap E_j) = A \cap \phi = \phi$ for $i \neq j$. So these parts are disjoint.
- Step 4** By the axiom of union: $P(A) = P(A \cap E_1) + P(A \cap E_2) + \dots + P(A \cap E_n)$.
- Step 5** By the multiplication theorem: $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \dots + P(E_n)P(A/E_n)$... (2)
- Step 6** By the definition of conditional probability: $P(E_k/A) = \frac{P(E_k \cap A)}{P(A)} = \frac{P(E_k)P(A/E_k)}{P(A)}$.
- Step 7** Put (2) in the denominator. This gives the result. ■

EXAM METHOD — THE BAYES TABLE (FAST AND SAFE)

Source E_i	Prior $P(E_i)$	$P(A/E_i)$	Product $P(E_i)P(A/E_i)$	Posterior = Product ÷ Total
E_1	...	...	...	...
Total = $P(A)$			sum	sum = 1 ✓

The posteriors must add to 1. Use this as a check.

PYQ. In a bolt factory, machines A, B, C make 20%, 30%, 50% of the output. Of their output 6%, 3%, 2% are defective. A bolt is drawn and found defective. Find the probability that it is from A, B, C.

[Class notes](#)
[Oct/Nov-2021 \(similar\)](#)

Machine	$P(E_i)$	$P(D/E_i)$	Product	$P(E_i/D)$
A	0.2	0.06	0.012	$0.012/0.031 = 12/31$
B	0.3	0.03	0.009	$9/31$
C	0.5	0.02	0.010	$10/31$
Total $P(D)$			0.031	1 ✓

PYQ. A businessman goes to hotels X, Y, Z 20%, 50%, 30% of the time. 5%, 4%, 8% of the rooms in X, Y, Z have faulty plumbing. Find the probability that a room with faulty plumbing is in hotel Z.

[Nov-2024 · 10M](#)
[Mar/Apr-2022 \(hotel Y\)](#)
[Class notes](#)

Let F = faulty plumbing.

Hotel	$P(\cdot)$	$P(F/\cdot)$	Product	Posterior
X	0.20	0.05	0.010	$10/54 = 5/27$
Y	0.50	0.04	0.020	$20/54 = 10/27$
Z	0.30	0.08	0.024	$24/54 = 4/9$
$P(F)$			0.054	1 ✓

$$P(Z/F) = \frac{0.3 \times 0.08}{0.054} = \frac{4}{9} \approx 0.444. \quad \text{P(Z/F) = 4/9} \quad \text{P(Y/F) = 10/27}$$

PYQ. Of three men, the chances that a politician, a businessman or an academician will be appointed vice-chancellor are 0.5, 0.3, 0.2. The probabilities that research is promoted by them are 0.3, 0.7, 0.8. (a) Find the probability that research is promoted. (b) If research is promoted, find the probability that the VC is an academician. [Mar/Apr-2024 · 10M](#) [Nov-2020](#) [Class notes](#)

Let R = research is promoted.

Person	Prior	$P(R/\cdot)$	Product	Posterior
Politician (A)	0.5	0.3	0.15	$15/52 \approx 0.288$
Businessman (B)	0.3	0.7	0.21	$21/52 \approx 0.404$
Academician (C)	0.2	0.8	0.16	$16/52 = 4/13 \approx 0.308$
$P(R)$			0.52	1 ✓

$$(a) P(R) = 0.52 = 13/25 \quad (b) P(C/R) = 4/13$$

PYQ. Three companies X, Y, Z produce TVs. X produces twice as many as Y. Y and Z produce the same number. 2% of X, 2% of Y and 4% of Z are defective. A TV chosen at random is defective. Find the probability that it is produced by company Z, and by company X. [Apr/May-2023 · 10M](#) [Class notes](#)

$$P(X) = 2P(Y), P(Y) = P(Z). \text{ So } 2P(Y) + P(Y) + P(Y) = 1 \Rightarrow P(Y) = \frac{1}{4}, P(X) = \frac{1}{2}, P(Z) = \frac{1}{4}.$$

$$P(D) = \frac{1}{2}(0.02) + \frac{1}{4}(0.02) + \frac{1}{4}(0.04) = 0.01 + 0.005 + 0.01 = 0.025.$$

$$P(Z/D) = \frac{0.01}{0.025} = \frac{2}{5}; \quad P(X/D) = \frac{0.01}{0.025} = \frac{2}{5}. \quad \boxed{P(Z/D) = P(X/D) = 2/5}$$

CAUTION

One TV comes from one company only. In your class notes the two answers are multiplied ($\frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$). In the exam, give both answers $\frac{2}{5}$ separately. If the examiner means "Z or X", add them: $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$.

PYQ. A slip is given to X, who marks it with a plus sign with probability $1/3$, else a minus sign. X passes it to Y, Y to Z, Z to W, W to a referee. Each of Y, Z, W changes the sign with probability $2/3$. The referee sees a plus. Find the probability that X originally wrote a plus. Oct-2023 · 10M

Step 1 The sign stays the same if the number of changes is even (0 or 2). Let $c = \frac{2}{3}$ (change), $k = \frac{1}{3}$ (keep).

$$P(\text{even}) = k^3 + {}^3C_2 c^2 k = \frac{1}{27} + 3 \cdot \frac{4}{9} \cdot \frac{1}{3} = \frac{1}{27} + \frac{12}{27} = \frac{13}{27}. \quad P(\text{odd}) = \frac{14}{27}.$$

Step 2 Let $E_1 = X$ wrote plus ($\frac{1}{3}$), $E_2 = X$ wrote minus ($\frac{2}{3}$), $A =$ referee sees plus.

$$P(A/E_1) = \frac{13}{27} \text{ (even changes)}, \quad P(A/E_2) = \frac{14}{27} \text{ (odd changes)}.$$

Step 3 Bayes:
$$P(E_1/A) = \frac{\frac{1}{3} \cdot \frac{13}{27}}{\frac{1}{3} \cdot \frac{13}{27} + \frac{2}{3} \cdot \frac{14}{27}} = \frac{13}{13 + 28} = \frac{13}{41}.$$

13/41

Example 13. In a college, 25% of boys and 10% of girls study mathematics. Girls are 60% of the students. (i) Find $P(\text{mathematics})$. (ii) A mathematics student is selected. Find the probability that the student is a girl, and a boy. Class notes

$$P(B) = 0.4, \quad P(G) = 0.6, \quad P(M/B) = 0.25, \quad P(M/G) = 0.10.$$

$$(i) \quad P(M) = 0.4(0.25) + 0.6(0.10) = 0.10 + 0.06 = 0.16 = \frac{4}{25}.$$

$$(ii) \quad P(G/M) = \frac{0.06}{0.16} = \frac{3}{8} = 0.375; \quad P(B/M) = \frac{0.10}{0.16} = \frac{5}{8} = 0.625.$$

Example 14. 5 men in 100 and 25 women in 10,000 are colour-blind. A colour-blind person is selected. Find the probability that the person is male (assume equal numbers of males and females). Class notes

$$P(M) = P(W) = \frac{1}{2}, \quad P(B/M) = \frac{5}{100}, \quad P(B/W) = \frac{25}{10000}.$$

$$P(M/B) = \frac{\frac{1}{2} \cdot \frac{5}{100}}{\frac{1}{2} \cdot \frac{5}{100} + \frac{1}{2} \cdot \frac{25}{10000}} = \frac{500}{500 + 25} = \frac{20}{21}. \quad P(W/B) = \frac{1}{21}.$$

1.10 Quick Revision Sheet

Basic

$$P(E) = \frac{n(E)}{n(S)}, \quad 0 \leq P(E) \leq 1$$

$$P(\bar{E}) = 1 - P(E)$$

$$\text{At least one} = 1 - P(\text{none})$$

Axioms

$$(1) P(E) \geq 0 \quad (2) P(S) = 1$$

$$(3) A \cap B = \phi \Rightarrow P(A \cup B) = P(A) + P(B)$$

Addition

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$

Conditional / multiplication

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A)P(B/A)$$

$$\text{Independent: } P(A \cap B) = P(A)P(B)$$

Total probability

$$P(A) = \sum P(E_i)P(A/E_i)$$

Bayes

$$P(E_k/A) = \frac{P(E_k)P(A/E_k)}{\sum P(E_i)P(A/E_i)}$$

Alternate turns (A starts)

$$P(A) = \frac{p_1}{1 - q_1 q_2}$$

Counting

$${}^n C_r = \frac{n!}{r!(n-r)!}; \text{ two dice } n(S) = 36; n \text{ coins}$$

$$n(S) = 2^n; \text{ cards } {}^{52} C_r$$

★ Assignment — Unit 1 (5 questions)

1. State the axioms of probability. Prove that $P(\bar{A}) = 1 - P(A)$ and $P(\phi) = 0$. [5M]
2. A bag has 6 white and 4 black balls. Three balls are drawn one by one without replacement. Find the probability that they are drawn in the order white, black, white. [5M]
3. A and B throw a die alternately. The first to throw a 6 wins. A starts. Find the probability that (i) A wins (ii) B wins. [5M]
4. State and prove Bayes' theorem. [10M]
5. Three urns contain 2 white + 3 black, 3 white + 2 black, and 4 white + 1 black balls. One urn is chosen at random and one ball is drawn. It is white. Find the probability that it came from the first urn. [10M]

Answers for self-check: Q2: $\frac{6}{10} \cdot \frac{4}{9} \cdot \frac{5}{8} = \frac{1}{6}$. Q3: $P(A) = \frac{1/6}{1-25/36} = \frac{6}{11}$, $P(B) = \frac{5}{11}$. Q5: $P(W) = \frac{1}{3} \left(\frac{2}{5} + \frac{3}{5} + \frac{4}{5} \right) = \frac{3}{5}$;
 $P(U_1/W) = \frac{2/15}{3/5} = \frac{2}{9}$.