

Unit 2 — Random Variables & Distributions

Discrete & continuous RVs · Distribution function · Binomial · Poisson · Normal · Sampling distribution · Standard error

Exam slot: Part-B Q4 / Q5 (10 M)

Part-A: 1c, 1d (2 + 3 M)

Written in ASD-STE100 style

PYQ TREND (2020 – 2026): WHAT THE EXAM ASKS FROM THIS UNIT

- **Normal distribution** — "how many students" or **find μ and σ from percentages**: asked in almost every paper.
- **Population of 5 or 6 numbers** — **all samples of size 2 or 3** (mean, s.d., mean and s.d. of sampling distribution): May-2025, Nov-2024, Oct-2023, Oct-2021, Nov-2020.
- **Continuous PDF with k** — find k, probabilities, mean: May-2025, Nov-2020; $f(x) = y_0 e^{-|x|}$ in Part A (Apr-2026, Nov-2024).
- **Binomial from mean and variance; Poisson** (rare events, $P(1)=P(2)$, cars hired out).
- **Short questions**: state Poisson / Binomial, properties of Normal, discrete vs continuous, standard error.

2.1 Random variable & types

2.2 Discrete RV: PMF, CDF, mean, variance

2.3 Continuous RV: PDF, mean, variance, mode, median

2.4 Properties of expectation

2.5 Binomial distribution

2.6 Poisson distribution

2.7 Normal distribution

2.8 Comparison of B, P, N

2.9 Sampling distribution & standard error

2.10 Revision sheet + Normal table

★ Assignment (5 questions)

2.1 Random Variable and its Types

RANDOM VARIABLE (RV)

A **random variable** X is a real-valued function on the sample space S of a random experiment. The outcome of the experiment gives the value of X . For each value x , $P(X = x)$ is the probability of the event that X represents.

Example 1. Give two examples of a random variable. [Class notes](#)

- Toss two coins. $S = \{TT, TH, HT, HH\}$. Let X = no. of heads. Then $X \in \{0, 1, 2\}$.
- Roll a pair of dice. Let $X(a, b) = \min\{a, b\}$. Then $X \in \{1, 2, 3, 4, 5, 6\}$.

	Discrete random variable	Continuous random variable
Values	Only separate (countable) values, like $0, 1, 2, \dots, n$.	All values in an interval.
Probability function	Probability mass function (PMF) $p(x) = P(X = x)$	Probability density function (PDF) $f(x)$; $P(X = a) = 0$
Total probability	$\sum p(x_i) = 1$	$\int_{-\infty}^{\infty} f(x) dx = 1$
Mean	$\mu = \sum x_i p_i$	$\mu = \int x f(x) dx$
Examples	No. of heads, no. of defective bolts, no. of calls	Height, weight, time, temperature

PYQ: "Distinguish between discrete and continuous random variables with examples." **Oct-2025 · 2M** — use this table.

2.2 Discrete Random Variable

PROBABILITY MASS FUNCTION (PMF) AND DISTRIBUTION

Let X be a discrete RV with values $x_1, x_2, \dots, x_n$ and $p_i = P(X = x_i)$. The function $p(x)$ is a **probability mass function** if:

(1) $p(x_i) \geq 0$ for all i (2) $\sum_{i=1}^n p(x_i) = 1$.

The set $\{(x_i, p_i)\}$ is the **discrete probability distribution** of X . Write it as a table:

$X = x_i$	x_1	x_2	x_3	...	x_n
$P(x_i)$	p_1	p_2	p_3	...	p_n

DISTRIBUTION FUNCTION (CDF), MEAN, VARIANCE

Distribution function: $F(x) = P(X \leq x) = \sum_{x_i \leq x} p(x_i)$. Also: $P(X < x_i) = P(X \leq x_{i-1})$, $P(X \geq x_i) = 1 - P(X < x_i)$, $P(X > x_i) = 1 - P(X \leq x_i)$.

Mean (expectation): $\mu = E(X) = \sum x_i p_i$; $E[g(X)] = \sum g(x_i) p_i$.

Variance: $\sigma^2 = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2 = E(X^2) - [E(X)]^2$.

Standard deviation: $\sigma = +\sqrt{\sum x_i^2 p_i - \mu^2}$.

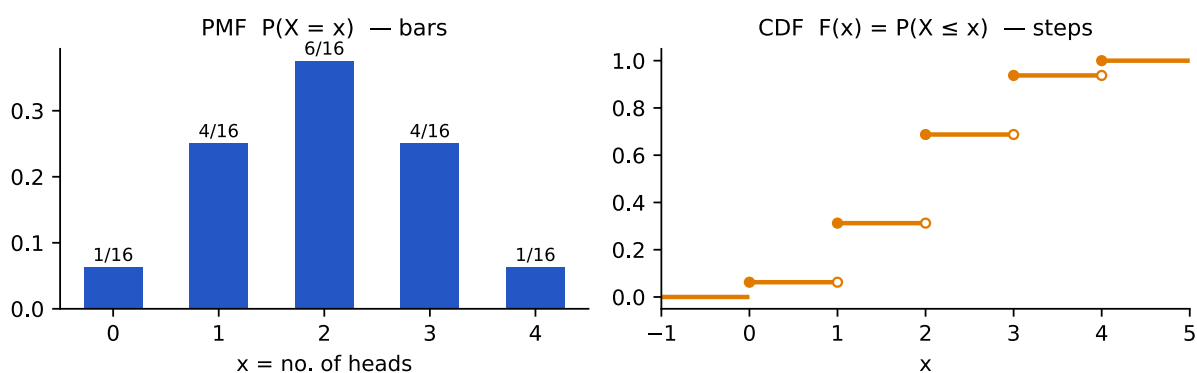


Fig 2.1 — X = no. of heads in 4 coins. The PMF is bars. The CDF is a step function that rises to 1.

Example 2. X = no. of heads in a single toss of 4 fair coins. Find (i) $P(X < 2)$ (ii) $P(1 < X \leq 3)$ (iii) $P(X \geq 1)$ (iv) $P(X \leq 3)$. [Class notes](#)

$$n(S) = 2^4 = 16. P(X = r) = {}^4C_r / 16.$$

x	0	1	2	3	4
$P(x)$	1/16	4/16	6/16	4/16	1/16

(i) $P(0) + P(1) = \frac{5}{16}$ (ii) $P(2) + P(3) = \frac{10}{16} = \frac{5}{8}$ (iii) $1 - P(0) = \frac{15}{16}$ (iv) $1 - P(4) = \frac{15}{16}$

PYQ. A pair of fair dice is rolled. Let X = maximum of the two numbers. Find the probability distribution of X , its mean and variance. Oct-2023 · 3M Class notes

$n(S) = 36$. $X = k$ when the larger number is k : there are $2k - 1$ such pairs. So $P(X = k) = \frac{2k - 1}{36}$.

x	1	2	3	4	5	6
$P(x)$	1/36	3/36	5/36	7/36	9/36	11/36

Mean $\mu = \frac{1+6+15+28+45+66}{36} = \frac{161}{36} = 4.472$. $\sum x^2 p = \frac{791}{36} = 21.972$.
 $\sigma^2 = 21.972 - (4.472)^2 = 1.971$; $\sigma = 1.404$.

PYQ. A die is rolled twice. X = minimum of the two numbers. Find the distribution of X , its mean and variance. Nov/Dec-2022 · 10M Class notes

$P(X = k) = \frac{13 - 2k}{36}$:

x	1	2	3	4	5	6
$P(x)$	11/36	9/36	7/36	5/36	3/36	1/36

$\mu = \frac{11+18+21+20+15+6}{36} = \frac{91}{36} = 2.528$. $\sum x^2 p = \frac{11+36+63+80+75+36}{36} = \frac{301}{36}$.
 $\sigma^2 = \frac{301}{36} - \left(\frac{91}{36}\right)^2 = 1.971$; $\sigma = 1.404$. $\mu = 2.528, \sigma^2 = 1.971$

Example 3. Two dice are thrown. $X(a, b) = a + b$. Find the distribution, mean, variance and s.d.

Class notes

x	2	3	4	5	6	7	8	9	10	11	12
$36P(x)$	1	2	3	4	5	6	5	4	3	2	1

$\mu = \frac{2+6+12+20+30+42+40+36+30+22+12}{36} = \frac{252}{36} = 7$. $\sum x^2 p = \frac{1974}{36} = 54.833$.
 $\sigma^2 = 54.833 - 49 = 5.833$; $\sigma = 2.415$.

CORRECTION TO CLASS NOTES

The class notes miss the last term $12 \times \frac{1}{36}$ and get $\mu = \frac{240}{36}$. The correct mean is $\frac{252}{36} = 7$, and $\sigma^2 = \frac{35}{6} = 5.833$.

Example 4. X has the distribution below. (i) Find k . (ii) Find $P(X < 6)$, $P(X \geq 6)$, $P(0 < X < 5)$. (iii) Find the minimum a such that $P(X \leq a) > \frac{1}{2}$. (iv) Find the mean and variance. Class notes

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i) $\sum P = 1 \Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow (10k - 1)(k + 1) = 0 \Rightarrow k = \frac{1}{10}$ ($k = -1$ is rejected because $P \geq 0$).
(ii) $P(X < 6) = 1 - [P(6) + P(7)] = 1 - \left[\frac{2}{100} + \frac{17}{100}\right] = \frac{81}{100}$; $P(X \geq 6) = \frac{19}{100}$; $P(0 < X < 5) = \frac{8}{10}$.

(iii) $P(X \leq 3) = \frac{5}{10}$ (not $> \frac{1}{2}$); $P(X \leq 4) = \frac{8}{10} > \frac{1}{2}$. So the minimum value is 4.

(iv) $\mu = \sum xp = 3.66$; $\sum x^2p = 16.80$; $\sigma^2 = 16.80 - 3.66^2 = 3.4044$.

PYQ. A person wins Rs. 80 if 3 heads occur, Rs. 30 if 2 heads, Rs. 10 if only 1 head, in a single toss of 3 fair coins. If the game is fair, how much must he lose if no heads occur? Mar/Apr-2024 · 3M

Let he lose Rs. a . A game is **fair** when the expected gain $E(X) = 0$.

No. of heads	3	2	1	0
Gain x	80	30	10	$-a$
$P(x)$	$1/8$	$3/8$	$3/8$	$1/8$

$$E(X) = \frac{80+90+30-a}{8} = 0 \Rightarrow a = 200. \quad \text{He must lose Rs. 200}$$

Example 5. A player tosses 3 fair coins. He wins Rs. 500 for 3 heads, Rs. 200 for 2 heads, Rs. 100 for 1 head. He loses Rs. 1500 for 3 tails. Find his expected gain. Class notes

$E(X) = 500 \cdot \frac{1}{8} + 200 \cdot \frac{3}{8} + 100 \cdot \frac{3}{8} - 1500 \cdot \frac{1}{8} = \frac{500+600+300-1500}{8} = -\frac{25}{2}$. The expected gain is negative, so the game is **not favourable**.

Example 6. A sample of 3 items is selected from a box of 10 items, of which 4 are defective. Find the expected number of defective items. Class notes

$$P(X = r) = \frac{{}^4C_r \cdot {}^6C_{3-r}}{{}^{10}C_3}; P(0) = \frac{1}{6}, P(1) = \frac{1}{2}, P(2) = \frac{3}{10}, P(3) = \frac{1}{30}.$$

$$E(X) = 0 + \frac{1}{2} + \frac{6}{10} + \frac{3}{30} = \frac{6}{5} = 1.2.$$

PYQ. A random variable X takes the values $-2, -1, 0, 1, 2$ such that $P(X = 0) = P(X < 0) = P(X > 0)$ (equal probabilities on each side). Find the probability distribution and the distribution function of X . Jul-2021 · 7M

$P(X < 0) + P(X = 0) + P(X > 0) = 1$, so each is $\frac{1}{3}$. Split each side equally: $P(-2) = P(-1) = \frac{1}{6}$, $P(1) = P(2) = \frac{1}{6}$.

x	-2	-1	0	1	2
$P(x)$	$1/6$	$1/6$	$1/3$	$1/6$	$1/6$
$F(x)$	$1/6$	$2/6$	$4/6$	$5/6$	1

2.3 Continuous Random Variable

PROBABILITY DENSITY FUNCTION (PDF)

For a continuous RV X , $f(x)$ is the **probability density function** if $P\left(x - \frac{dx}{2} \leq X \leq x + \frac{dx}{2}\right) = f(x) dx$.

The curve $y = f(x)$ is the **probability curve**.

Properties: (1) $f(x) \geq 0$ (2) $\int_{-\infty}^{\infty} f(x) dx = 1$ (3) $P(a \leq X \leq b) = \int_a^b f(x) dx = \text{area under the curve from } a \text{ to } b$.

FORMULAS — CONTINUOUS RV

$$P(X > a) = \int_a^{\infty} f(x)dx, \quad P(X < b) = \int_{-\infty}^b f(x)dx$$

Distribution function: $F(x) = \int_{-\infty}^x f(t)dt$; $f(x) = F'(x)$

Mean: $\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx$

Variance: $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$

Mode: value x_0 where $f'(x_0) = 0$ and $f''(x_0) < 0$

Median M : $\int_{-\infty}^M f(x)dx = \frac{1}{2}$

Mean deviation: $\int |x - \mu| f(x)dx$

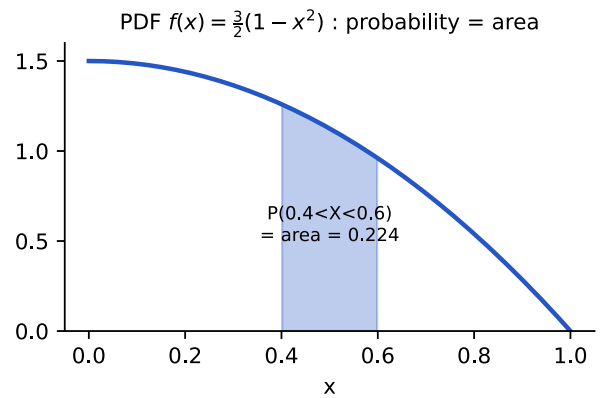


Fig 2.2 — For a PDF, probability = area under the curve.

NOTE — USEFUL INTEGRALS

$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$. $\int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx$ if f is even; $= 0$ if f is odd. Integration by parts:
 $\int uv dx = u \int v dx - \int (u' \int v dx) dx$.

PYQ. A continuous RV X has PDF $f(x) = k(1 - x^2)$ for $0 < x < 1$, and 0 elsewhere. (a) Find k and $P(X > 0.5)$. (b) Using the distribution function, find $P(X < 0.3)$ and $P(0.4 < X < 0.6)$. (c) Find the mean. May-2025 · 10M Class notes

Step 1 $\int_0^1 k(1 - x^2)dx = 1 \Rightarrow k \left[x - \frac{x^3}{3} \right]_0^1 = k \cdot \frac{2}{3} = 1 \Rightarrow k = \frac{3}{2}$.

Step 2 $P(X > 0.5) = \frac{3}{2} \left[x - \frac{x^3}{3} \right]_{0.5}^1 = \frac{3}{2} \left[\frac{2}{3} - \frac{11}{24} \right] = \frac{3}{2} \cdot \frac{5}{24} = \frac{5}{16} = 0.3125$.

Step 3 Distribution function: $F(x) = \int_0^x \frac{3}{2}(1 - t^2)dt = \frac{3}{2} \left(x - \frac{x^3}{3} \right)$, $0 \leq x \leq 1$.

Step 4 $P(X < 0.3) = F(0.3) = \frac{3}{2}(0.3 - 0.009) = 0.4365$.
 $P(0.4 < X < 0.6) = F(0.6) - F(0.4) = \frac{3}{2}(0.528) - \frac{3}{2}(0.37867) = 0.224$.

Step 5 Mean $= \int_0^1 x \cdot \frac{3}{2}(1 - x^2)dx = \frac{3}{2} \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{3}{2} \cdot \frac{1}{4} = \frac{3}{8}$.

$k = 3/2 \cdot P(X > 0.5) = 0.3125 \cdot 0.4365 \cdot 0.224 \cdot \mu = 3/8$

Class notes also find $P(0.1 < X < 0.2) = \frac{3}{2} \left[0.1 - \frac{0.007}{3} \right] = 0.1465$.

PYQ. If $f(x) = y_0 e^{-|x|}$, $-\infty < x < \infty$, is a PDF, find y_0 . (Class notes also: find mean, variance and $P(0 < X < 4)$.) Apr/May-2026 · 3M Nov-2024 · 3M Class notes

$e^{-|x|}$ is an even function. So $\int_{-\infty}^{\infty} y_0 e^{-|x|} dx = 2y_0 \int_0^{\infty} e^{-x} dx = 2y_0 [-e^{-x}]_0^{\infty} = 2y_0(0 + 1) = 1 \Rightarrow y_0 = \frac{1}{2}$.

Mean: $\int x \cdot \frac{1}{2} e^{-|x|} dx = 0$ (odd function). **Variance:** $\sigma^2 = 2 \cdot \frac{1}{2} \int_0^{\infty} x^2 e^{-x} dx - 0 = \frac{2!}{1^3} = 2$.

$P(0 < X < 4) = \frac{1}{2} \int_0^4 e^{-x} dx = \frac{1}{2}(1 - e^{-4}) = 0.4908$. **$y_0 = 1/2, \mu = 0, \sigma^2 = 2$**

PYQ. Find the mean and variance of $f(x) = e^{-x}$, $x > 0$. Apr/May-2023 · 3M

$\mu = \int_0^{\infty} x e^{-x} dx = 1! = 1$. $E(X^2) = \int_0^{\infty} x^2 e^{-x} dx = 2! = 2$. $\sigma^2 = 2 - 1 = 1$. **$\mu = 1, \sigma^2 = 1$**

Example 7. $f(x) = 2e^{-2x}$ for $x > 0$. Find $P(1 \leq X \leq 3)$ and $P(X > 0.5)$. [Class notes](#)

$$P(1 \leq X \leq 3) = \int_1^3 2e^{-2x} dx = [-e^{-2x}]_1^3 = e^{-2} - e^{-6} = 0.1329. \quad P(X > 0.5) = [-e^{-2x}]_{0.5}^{\infty} = e^{-1} = 0.3679.$$

PYQ. The PDF of X is $f(x) = \frac{1}{2} \sin x$ for $0 \leq x \leq \pi$, and 0 elsewhere. Find the mean, mode and median. Also find $P(0 < X < \pi/2)$. [Mar/Apr-2022 · 9M](#) [Class notes](#)

Mean: $\mu = \frac{1}{2} \int_0^{\pi} x \sin x dx = \frac{1}{2} \left[-x \cos x \right]_0^{\pi} + \frac{1}{2} \int_0^{\pi} \cos x dx = \frac{1}{2}(\pi) + \frac{1}{2}[\sin x]_0^{\pi} = \frac{\pi}{2}.$

Mode: $f'(x) = \frac{1}{2} \cos x = 0 \Rightarrow x = \frac{\pi}{2}. f''(\frac{\pi}{2}) = -\frac{1}{2} \sin \frac{\pi}{2} = -\frac{1}{2} < 0.$ So mode $= \frac{\pi}{2}.$

Median: $\int_0^M \frac{1}{2} \sin x dx = \frac{1}{2} \Rightarrow \frac{1}{2}[1 - \cos M] = \frac{1}{2} \Rightarrow \cos M = 0 \Rightarrow M = \frac{\pi}{2}.$

$P(0 < X < \frac{\pi}{2}) = \frac{1}{2}[-\cos x]_0^{\pi/2} = \frac{1}{2}.$ **Mean = Mode = Median = $\pi/2$**

PYQ. A continuous RV has PDF $f(x) = kxe^{-\lambda x}$ for $x \geq 0, \lambda > 0$, and 0 otherwise. Find (i) k (ii) mean (iii) variance. [Nov-2020 · 7M](#) [Class notes](#)

Use $\int_0^{\infty} x^n e^{-\lambda x} dx = \frac{n!}{\lambda^{n+1}}.$

(i) $k \cdot \frac{1}{\lambda^2} = 1 \Rightarrow k = \lambda^2.$ (ii) $\mu = \lambda^2 \int_0^{\infty} x^2 e^{-\lambda x} dx = \lambda^2 \cdot \frac{2}{\lambda^3} = \frac{2}{\lambda}.$

(iii) $E(X^2) = \lambda^2 \cdot \frac{3!}{\lambda^4} = \frac{6}{\lambda^2}; \sigma^2 = \frac{6}{\lambda^2} - \frac{4}{\lambda^2} = \frac{2}{\lambda^2}.$

Example 8. $f(x) = \frac{1}{16}(3+x)^2$ for $-3 \leq x < -1$; $\frac{1}{16}(6-2x^2)$ for $-1 \leq x < 1$; $\frac{1}{16}(3-x)^2$ for $1 \leq x \leq 3$; 0 elsewhere. Show that f is a PDF and find the mean. [Class notes](#)

$\int_{-3}^{-1} \frac{(3+x)^2}{16} dx = \frac{1}{16} \cdot \frac{8}{3}, \int_{-1}^1 \frac{6-2x^2}{16} dx = \frac{1}{16} \cdot \frac{32}{3}, \int_1^3 \frac{(3-x)^2}{16} dx = \frac{1}{16} \cdot \frac{8}{3}.$ Sum $= \frac{1}{16} \cdot \frac{48}{3} = 1 \checkmark.$
 $f(-x) = f(x)$ (symmetric about 0), so $xf(x)$ is odd. Mean = 0.

2.4 Properties of Expectation and Variance

RESULTS (A, B, K, L ARE CONSTANTS)

(1) $E(aX + b) = aE(X) + b$ (2) $Var(aX + b) = a^2 Var(X)$ (3) $E(kX + lY) = kE(X) + lE(Y)$ (4) $Var(b) = 0$

PYQ. If X is a continuous RV and $Y = aX + b$, prove that $E(Y) = aE(X) + b$ and $Var(Y) = a^2 Var(X)$.

[Nov/Dec-2022 · 3M](#)

$E(aX + b) = \int (ax + b)f(x)dx = a \int xf(x)dx + b \int f(x)dx = aE(X) + b \cdot 1.$

$Var(Y) = E[(Y - E(Y))^2] = E[(aX + b - aE(X) - b)^2] = E[a^2(X - E(X))^2] = a^2 E[(X - \mu)^2] = a^2 Var(X). \blacksquare$

Example 9. $X: -3, 6, 9$ with $P = \frac{1}{6}, \frac{1}{2}, \frac{1}{3}$. Find $E(X), E(X^2), E[(2X + 1)^2]$. [Class notes](#)

$E(X) = -\frac{3}{6} + 3 + 3 = \frac{11}{2}. E(X^2) = \frac{9}{6} + 18 + 27 = \frac{93}{2}. E[(2X + 1)^2] = 4E(X^2) + 4E(X) + 1 = 186 + 22 + 1 = 209.$

2.5 Binomial Distribution

DEFINITION OCT-2023 · 2M (STATE)

A random variable X has a **binomial distribution** if it takes only non-negative values and its probability function is

$$P(X = r) = p(r) = {}^nC_r p^r q^{n-r}, \quad r = 0, 1, 2, \dots, n; \quad q = 1 - p$$

Here n = no. of trials, p = probability of success in one trial. We write $X \sim B(n, p)$.

Examples: no. of defective bolts in a bag of n bolts; no. of heads in n tosses.

CONDITIONS FOR THE BINOMIAL DISTRIBUTION

(1) The trials are repeated a fixed number of times, n . (2) Each trial has only 2 outcomes: success or failure. (3) The probability of success p stays constant from trial to trial. (4) The trials are independent.

MEAN AND VARIANCE

$$\mu = np \quad \sigma^2 = npq \quad \sigma = \sqrt{npq} \quad (\text{variance} < \text{mean, because } q < 1)$$

$$\text{Recurrence: } P(r+1) = \frac{n-r}{r+1} \cdot \frac{p}{q} \cdot P(r)$$

Derivation of mean, $\mu = np$

$$\begin{aligned} \mu &= \sum_{r=0}^n r {}^nC_r p^r q^{n-r} = 1 \cdot {}^nC_1 p q^{n-1} + 2 \cdot {}^nC_2 p^2 q^{n-2} + \dots + n p^n \\ &= np \left[q^{n-1} + (n-1)p q^{n-2} + \frac{(n-1)(n-2)}{2!} p^2 q^{n-3} + \dots + p^{n-1} \right] = np (q+p)^{n-1} = np (1)^{n-1} = np. \end{aligned}$$

$$\text{Variance: } \sigma^2 = \sum r(r-1)p(r) + \sum r p(r) - \mu^2 = n(n-1)p^2 + np - n^2 p^2 = np(1-p) = npq.$$

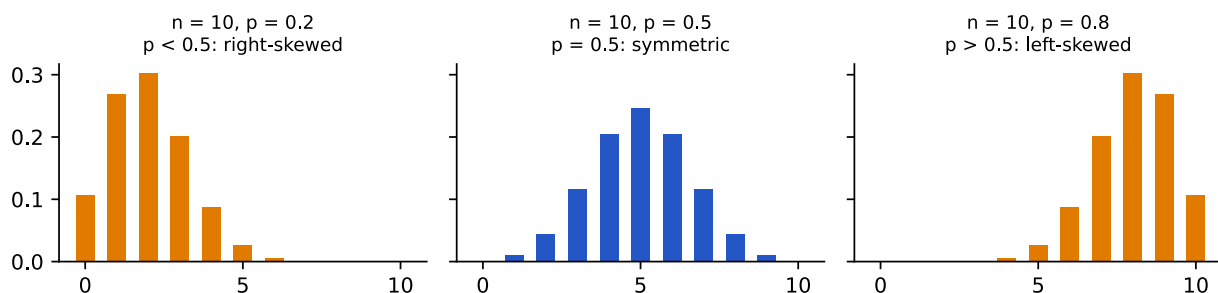


Fig 2.3 — Shape of the binomial distribution depends on p .

PYQ. $X \sim B(n = 10, p = 0.5)$. Find $P(X = 6)$. Oct-2025 · 5M

$$P(6) = {}^{10}C_6 (0.5)^{10} = \frac{210}{1024} = 0.2051.$$

PYQ. The mean of a binomial distribution is 3 and the variance is $9/4$. Find (i) n (ii) $P(1 \leq X < 6)$.

Nov-2024 · 5M

$$np = 3, npq = \frac{9}{4} \Rightarrow q = \frac{9/4}{3} = \frac{3}{4}, p = \frac{1}{4}, n = \frac{3}{1/4} = 12.$$

$$P(1 \leq X < 6) = \sum_{r=1}^5 {}^{12}C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{12-r} = 0.9139. \quad \text{n = 12, P = 0.9139}$$

PYQ. In a binomial distribution of 5 independent trials, the probabilities of 1 and 2 successes are 0.4096 and 0.2048. Find the parameter p , mean and variance. Mar/Apr-2022 · 6M Class notes

$$\frac{P(2)}{P(1)} = \frac{{}^5C_2 p^2 q^3}{{}^5C_1 p q^4} = \frac{2p}{q} = \frac{0.2048}{0.4096} = \frac{1}{2} \Rightarrow 4p = q = 1 - p \Rightarrow p = \frac{1}{5}.$$

$$\text{Mean} = np = 1; \text{variance} = npq = 5 \cdot \frac{1}{5} \cdot \frac{4}{5} = 0.8.$$

PYQ. The probability of a man hitting a target is $1/3$. (a) If he fires 5 times, find the probability of at least 2 hits. (b) How many times must he fire so that the probability of at least one hit is more than 90%? Mar/Apr-2024 · 10M Class notes

$$(a) P(X \geq 2) = 1 - P(0) - P(1) = 1 - \left(\frac{2}{3}\right)^5 - 5 \cdot \frac{1}{3} \left(\frac{2}{3}\right)^4 = 1 - \frac{32}{243} - \frac{80}{243} = \frac{131}{243} = 0.5391.$$

$$(b) 1 - \left(\frac{2}{3}\right)^n > 0.9 \Rightarrow \left(\frac{2}{3}\right)^n < 0.1. \text{ Try: } n = 5: 0.132; n = 6: 0.088 \checkmark. \quad \mathbf{n = 6}$$

PYQ. Out of 800 families with 5 children each, how many would you expect to have (i) 3 boys (ii) 5 girls (iii) 2 or 3 boys (iv) at least one boy? Assume equal probabilities for boys and girls. Nov-2020 · 8M Class notes

$$n = 5, p = q = \frac{1}{2}, P(r) = \frac{{}^5C_r}{32}. \text{ Expected no.} = N \cdot P \text{ with } N = 800.$$

$$(i) 800 \cdot \frac{10}{32} = 250 \quad (ii) 800 \cdot \frac{1}{32} = 25 \quad (iii) 800 \cdot \frac{20}{32} = 500 \quad (iv) 800 \cdot \frac{31}{32} = 775$$

Example 10. More class problems (answers). Class notes

- Coin tossed 6 times, exactly 4 heads: ${}^6C_4/2^6 = \frac{15}{64}$.
- 10 coins: at least 7 heads = $\frac{176}{1024} = \frac{11}{64}$; at least 6 heads = $\frac{386}{1024} = \frac{193}{512}$; at least 1 head = $\frac{1023}{1024}$.
- Sum 6 exactly 3 times in 7 throws of two dice: $p = \frac{5}{36}$; ${}^7C_3 \left(\frac{5}{36}\right)^3 \left(\frac{31}{36}\right)^4 = 0.0516$.
- Solar heat, $p = 0.6$, 5 installations: exactly 4 = 0.2592; at least 4 = 0.3370.
- Occupational disease 20%, 6 workers, 4 or more = $\frac{53}{3125} = 0.01696$.
- Mean 4, variance $4/3$: $q = \frac{1}{3}$, $p = \frac{2}{3}$, $n = 6$; $P(X \geq 1) = 1 - \frac{1}{729} = \frac{728}{729}$.
- 18 students, $p = \frac{1}{2}$: exactly 10 = 0.1669; at least 10 = 0.4073; at most 8 = 0.4073; $2 \leq X \leq 9$: 0.5927.

2.6 Poisson Distribution

DEFINITION MAY-2025 · 2M NOV-2024 · 2M

A random variable X has a **Poisson distribution** if it takes only non-negative values and

$$P(X = r) = \frac{e^{-\lambda} \lambda^r}{r!}, \quad r = 0, 1, 2, \dots$$

$\lambda > 0$ is the **parameter**. Mean = λ , variance = λ .

Conditions (limiting case of Binomial)

- (1) X is discrete. (2) The events are rare: $p \rightarrow 0$.
 (3) The no. of trials is very large: $n \rightarrow \infty$. (4)
 $np = \lambda$ is finite.

Examples of rare events Apr/May-2026 · 2M

- (1) Defective bulbs made by a good company. (2) Printing mistakes per page. (3) Particles emitted by a radioactive source. (4) Persons born blind per year in a city. (5) Accidents per day at a junction. (6) Telephone calls per minute.

Derivation from the Binomial distribution

$$P(r) = {}^nC_r p^r q^{n-r} = \frac{n(n-1)\cdots(n-r+1)}{r!} p^r (1-p)^{n-r}. \text{ Put } p = \frac{\lambda}{n}:$$

$$P(r) = \frac{\lambda^r}{r!} \cdot 1 \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \cdots \left(1 - \frac{r-1}{n}\right) \cdot \frac{\left(1 - \frac{\lambda}{n}\right)^n}{\left(1 - \frac{\lambda}{n}\right)^{r}}.$$

Let $n \rightarrow \infty$. Each bracket $\rightarrow 1$, and $\lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^n = e^{-\lambda}$. So $P(r) = \frac{e^{-\lambda} \lambda^r}{r!}$. ■

Mean: $\mu = \sum_{r=0}^{\infty} r \frac{e^{-\lambda} \lambda^r}{r!} = \lambda e^{-\lambda} \sum_{r=1}^{\infty} \frac{\lambda^{r-1}}{(r-1)!} = \lambda e^{-\lambda} e^{\lambda} = \lambda$. **Variance:** $\sum r(r-1)p(r) + \lambda - \lambda^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$.

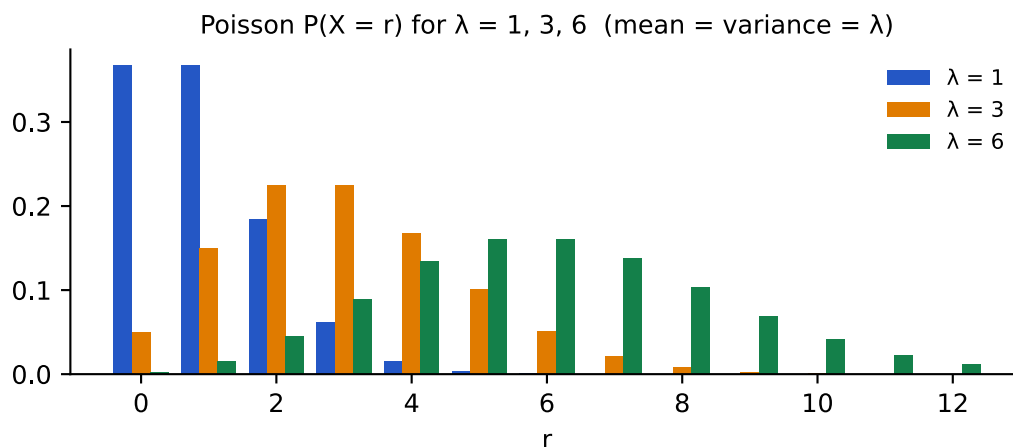


Fig 2.4 — Poisson probabilities for three values of λ .

PYQ. Estimate the probability that out of 2000 persons exactly 5 will suffer a bad reaction, if the probability of a bad reaction is 0.001. Mar/Apr-2024 · 5M Class notes (exactly 3, more than 2, none, more than 1)

$$\lambda = np = 2000 \times 0.001 = 2. \quad P(5) = \frac{e^{-2} 2^5}{5!} = \frac{32}{120} e^{-2} = 0.0361.$$

Class version: $P(3) = \frac{4}{3} e^{-2} = 0.1804$; $P(X > 2) = 1 - e^{-2}(1 + 2 + 2) = 0.3233$; $P(0) = e^{-2} = 0.1353$;
 $P(X > 1) = 1 - 3e^{-2} = 0.5940$.

(The class notes write 0.1604 for $P(3)$. The correct value is 0.1804.)

PYQ. A car hire firm has two cars. The number of demands per day is Poisson with mean 1.5. Find the proportion of days on which (i) neither car is used (ii) some demand is refused. Oct-2023 · 5M

$$\lambda = 1.5. \quad (i) \quad P(0) = e^{-1.5} = 0.2231.$$

$$(ii) \quad \text{A demand is refused when demand} > 2: P(X > 2) = 1 - e^{-1.5} \left[1 + 1.5 + \frac{1.5^2}{2} \right] = 1 -$$

$$0.2231(3.625) = 0.1912. \quad \approx 19\% \text{ of days}$$

PYQ. A Poisson variate has $P(X = 1) = P(X = 2)$. Find (i) the mean (ii) $P(X \geq 1)$ (iii) $P(1 \leq X \leq 4)$.

Mar/Apr-2022 · 7M

Class notes

$$e^{-\lambda} \lambda = \frac{e^{-\lambda} \lambda^2}{2} \Rightarrow \lambda = 2 \ (\lambda \neq 0). \text{ Mean} = 2.$$

$$P(X \geq 1) = 1 - e^{-2} = 0.8647. \quad P(1 \leq X \leq 4) = e^{-2} \left[2 + 2 + \frac{4}{3} + \frac{2}{3} \right] = 6e^{-2} = 0.8120.$$

$$\text{Class notes also: } P(4) = \frac{2}{3}e^{-2} = 0.0902; \quad P(1 < X < 4) = P(2) + P(3) = 0.4511.$$

PYQ. X is Poisson with mean 2. Find (i) $P(X = 2)$ (ii) $P(1 \leq X < 3)$ (iii) $P(X = 0)$.

Jul-2021 · 8M

$$(i) 2e^{-2} = 0.2707 \quad (ii) P(1) + P(2) = 4e^{-2} = 0.5413 \quad (iii) e^{-2} = 0.1353$$

PYQ. 10% of bean seeds do not germinate. Seeds are sold in packets of 200 with a guarantee of 90% germination. Find the probability that a packet violates the guarantee.

Apr/May-2023 · 5M

Class notes (5%)

The guarantee fails if more than $200 \times 0.10 = 20$ seeds do not germinate. $\lambda = np = 200 \times 0.10 = 20$.

$$P(X > 20) = 1 - \sum_{r=0}^{20} \frac{e^{-20} 20^r}{r!} = 0.4409.$$

$$\text{Class notes version (5\% fail): } \lambda = 10, \quad P(X > 20) = 1 - \sum_{r=0}^{20} \frac{e^{-10} 10^r}{r!} = 0.0016.$$

Example 11. More class problems (answers).

Class notes

- Hospital switchboard, $\lambda = 4$ calls per 10 min: at most 2 calls $= 13e^{-4} = 0.2381$; exactly 3 $= 0.1954$.
- Blades 5% defective, box of 100, guarantee ≤ 10 defective: $\lambda = 5$, $P(X > 10) = 0.0137$.
- 2% defective, box of 100: $\lambda = 2$, $P(2) = 0.2707$, $P(X \geq 3) = 0.3233$.
- Breakdowns $\lambda = 1.8$: none $= 0.1653$; one $= 0.2975$; at least one $= 1 - 0.1653 = 0.8347$. (Class notes write 0.6347 — arithmetic slip.)
- 2% defective tools, shipment of 400: $\lambda = 8$; 3% or more ($X \geq 12$) $= 0.1119$; 2% or less ($X \leq 8$) $= 0.5925$.
- $\frac{3}{2}P(1) = P(3) \Rightarrow \lambda = 3$: $P(X \geq 1) = 0.9502$, $P(X \leq 3) = 0.6472$, $P(2 \leq X \leq 5) = 0.7169$.

2.7 Normal Distribution ★★

DEFINITION APR/MAY-2023 · 2M

A continuous RV X has a **normal distribution** if its PDF is

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$$

μ (mean) and σ (s.d.) are the parameters. We write $X \sim N(\mu, \sigma^2)$. The total area under the normal curve is 1.

Normal curve: bell-shaped, symmetric about μ ; mean = median = mode

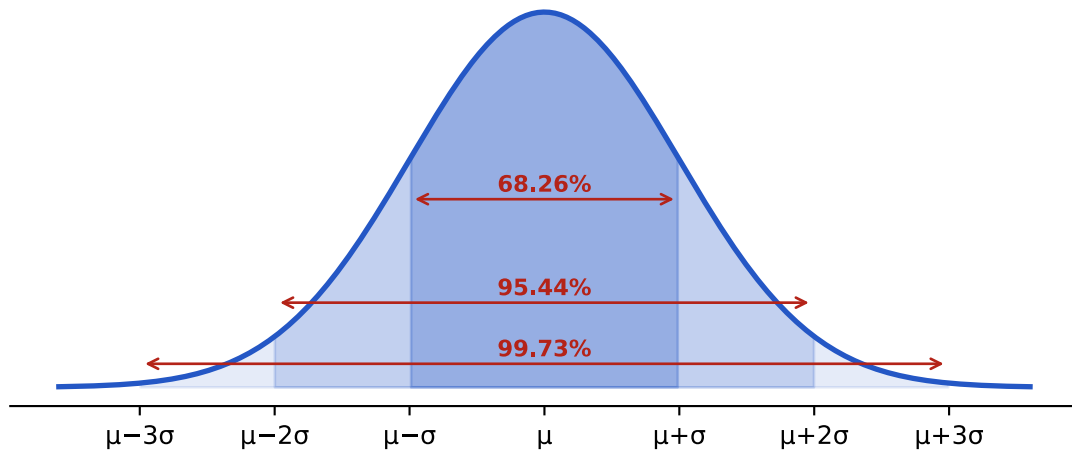


Fig 2.5 — Area under the normal curve within 1σ , 2σ , 3σ of the mean.

CHIEF CHARACTERISTICS (PROPERTIES) OF THE NORMAL CURVE OCT-2025 · 3M

1. The curve is bell-shaped and symmetric about the line $x = \mu$.
2. Mean = median = mode = μ . The curve is unimodal. The maximum height is $\frac{1}{\sigma\sqrt{2\pi}}$ at $x = \mu$.
3. The x -axis is an asymptote. The two tails extend to $\pm\infty$.
4. The total area under the curve is 1. Half of the area (0.5) is on each side of μ .
5. The points of inflection are at $x = \mu \pm \sigma$.
6. Area rule: $P(\mu - \sigma < X < \mu + \sigma) = 0.6826$, $P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9544$, $P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$.
7. Quartile deviation $\approx \frac{2}{3}\sigma$; mean deviation $\approx \frac{4}{5}\sigma$.
8. A linear combination of independent normal variates is also a normal variate.

STANDARD NORMAL VARIABLE Z

Put $Z = \frac{X - \mu}{\sigma}$. Then $Z \sim N(0, 1)$, with mean 0 and s.d. 1: $\phi(z) = \frac{1}{\sqrt{2\pi}}e^{-z^2/2}$. Now one table serves all normal curves.

$A(z_1)$ = area from $z = 0$ to $z = z_1 = P(0 \leq Z \leq z_1)$. Read it from the normal table (page at the end). Because of symmetry, $A(-z_1) = A(z_1)$.

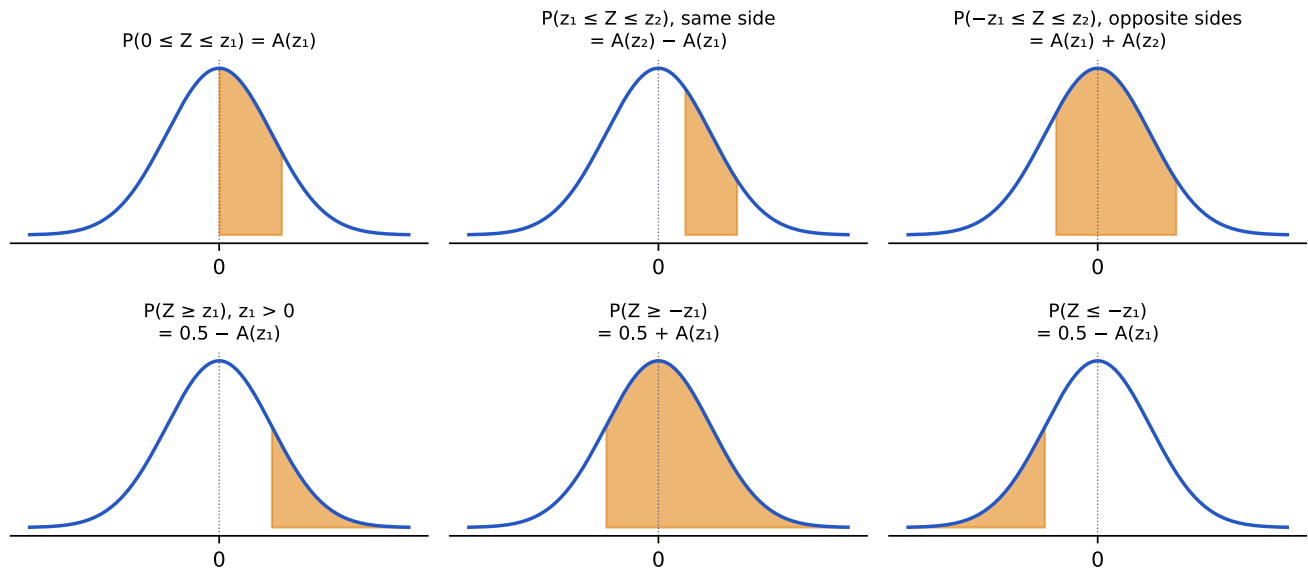


Fig 2.6 — The six area cases. Shaded area = required probability.

Method for normal problems

- Step 1** Write μ and σ . Write the required probability in terms of X .
- Step 2** Change each x value to $z = \frac{x - \mu}{\sigma}$.
- Step 3** Draw a small normal curve. Shade the required area.
- Step 4** Use the correct case from Fig 2.6. Read $A(z)$ from the table.
- Step 5** For "how many", multiply the probability by the total number N .

PYQ. Find the area under the standard normal curve (i) to the left of $z = -1.78$ (ii) between $z = -0.80$ and $z = 1.53$. May-2025 · 3M

- (i) $P(Z < -1.78) = 0.5 - A(1.78) = 0.5 - 0.4625 = 0.0375$.
 (ii) $A(0.80) + A(1.53) = 0.2881 + 0.4370 = 0.7251$.

PYQ. X is normal with mean 30 and s.d. 5. Find (i) $P(26 \leq X \leq 40)$ (ii) $P(|X - 30| > 5)$.

Nov-2024 · 5M Class notes

- (i) $z_1 = \frac{26-30}{5} = -0.8$, $z_2 = \frac{40-30}{5} = 2$. Opposite sides: $A(0.8) + A(2) = 0.2881 + 0.4772 = 0.7653$.
 (ii) $|X - 30| \leq 5 \iff 25 \leq X \leq 35 \iff -1 \leq Z \leq 1$: probability = $2A(1) = 0.6826$.

$P(|X - 30| > 5) = 1 - 0.6826 = 0.3174$. (i) 0.7653 (ii) 0.3174

Class notes also: $P(X \geq 45) = P(Z \geq 3) = 0.5 - 0.4987 = 0.0013$.

PYQ. For a normal variate with mean 1 and s.d. 3, find (i) $P(3.45 \leq X \leq 6.19)$ (ii) $P(-1.43 \leq X \leq 6.19)$.

Nov-2020 · 7M Class notes

- (i) $z_1 = \frac{3.45-1}{3} = 0.82$, $z_2 = \frac{6.19-1}{3} = 1.73$. Same side: $A(1.73) - A(0.82) = 0.4582 - 0.2939 = 0.1643$.
 (ii) $z_1 = \frac{-1.43-1}{3} = -0.81$: $A(0.81) + A(1.73) = 0.2910 + 0.4582 = 0.7492$.

PYQ. The masses of 300 students are normally distributed with mean 68 kg and s.d. 3 kg. How many students have masses (a) greater than 72 kg (b) less than or equal to 64 kg (c) between 65 and 71 kg inclusive? **Nov/Dec-2022 · 10M** [Class notes](#)

- (a) $z = \frac{72-68}{3} = 1.33$. $P = 0.5 - A(1.33) = 0.5 - 0.4082 = 0.0918$. No. = $300 \times 0.0918 = 27.5 \approx 28$.
 (b) $z = \frac{64-68}{3} = -1.33$. $P = 0.5 - 0.4082 = 0.0918$. No. ≈ 28 .
 (c) $z_1 = -1$, $z_2 = 1$. $P = 2(0.3413) = 0.6826$. No. = $300 \times 0.6826 = 204.8 \approx 205$.

PYQ. The marks of 1000 students are normally distributed with mean 78% and s.d. 11%. (a) How many students got marks above 90%? (b) What was the highest mark of the lowest 10%? (c) Within what limits did the middle 90% lie? **Apr/May-2023 · 10M** [Class notes](#)

- (a) $z = \frac{90-78}{11} = 1.09$. $P = 0.5 - A(1.09) = 0.5 - 0.3621 = 0.1379$. No. = $1000 \times 0.1379 \approx 138$.
 (b) Lowest 10%: area from z_1 to 0 is 0.4. Table: $z_1 = -1.28$. $x = 78 - 1.28(11) = 63.9 \approx 64\%$.
 (c) Middle 90%: 0.05 in each tail, so $A(z) = 0.45 \Rightarrow z = 1.645$. Limits = $78 \pm 1.645(11) = 78 \pm 18.1$.

$\approx 60\%$ to 96%

PYQ. The weights of 800 male students are normally distributed with mean 140 lb and s.d. 10 lb. Find the number of students whose weights are (i) between 138 and 148 lb (ii) more than 152 lb.

Mar/Apr-2022 · 8M [Class notes](#)

- (i) $z_1 = -0.2$, $z_2 = 0.8$: $A(0.2) + A(0.8) = 0.0793 + 0.2881 = 0.3674$. No. = $800 \times 0.3674 \approx 294$.
 (ii) $z = 1.2$: $0.5 - 0.3849 = 0.1151$. No. = $800 \times 0.1151 \approx 92$.

The 2022 paper prints $\mu = 100$. That is a misprint: with $\mu = 100$ the answer is almost 0. The standard problem (Schaum) uses $\mu = 140$.

Finding μ and σ from given percentages (frequent 5 M)

METHOD

- Step 1** Write each condition as $P(X < x_1) = a$ or $P(X > x_2) = b$.
Step 2 Find the area between 0 and each z : for example, $A(z_1) = 0.5 - a$.
Step 3 Read z from the table. Put a minus sign if the point is left of the mean.
Step 4 Write $\frac{x_1 - \mu}{\sigma} = z_1$ and $\frac{x_2 - \mu}{\sigma} = z_2$. Solve the two equations.

PYQ. Find the mean and s.d. of a normal distribution in which 7% of items are under 35 and 89% are under 63. **Mar/Apr-2024 · 5M** [Class notes](#)

$$A(z_1) = 0.5 - 0.07 = 0.43 \Rightarrow z_1 = -1.48 \text{ (left)}. \quad A(z_2) = 0.89 - 0.5 = 0.39 \Rightarrow z_2 = 1.23.$$

$$\frac{35 - \mu}{\sigma} = -1.48 \dots (1); \quad \frac{63 - \mu}{\sigma} = 1.23 \dots (2). \text{ Subtract: } \frac{28}{\sigma} = 2.71 \Rightarrow \sigma = 10.33; \quad \mu = 35 + 1.48(10.33) = 50.29.$$

$\mu \approx 50.3, \sigma \approx 10.3$

PYQ. In a normal distribution, 10.03% of items are under 25 kg and 89.97% are under 70 kg. Find the mean and s.d. Oct-2023 · 5M

$A(z_1) = 0.5 - 0.1003 = 0.3997 \Rightarrow z_1 = -1.28$. $A(z_2) = 0.8997 - 0.5 = 0.3997 \Rightarrow z_2 = 1.28$.
The two points are symmetric, so $\mu = \frac{25+70}{2} = 47.5$. $\frac{70-47.5}{\sigma} = 1.28 \Rightarrow \sigma = 17.58$.

$\mu = 47.5$ kg, $\sigma = 17.58$ kg

Example 12. (a) 31% of items are under 45 and 8% are over 64. (b) 10% are under 35 and 5% are above 90. Find μ and σ . Class notes

(a) $A(z_1) = 0.19 \Rightarrow z_1 = -0.5$; $A(z_2) = 0.42 \Rightarrow z_2 = 1.405$. Solve: $\mu = 50$, $\sigma = 10$, $\sigma^2 = 100$.
(b) $z_1 = -1.28$, $z_2 = 1.645$: $\sigma = \frac{55}{2.925} = 18.8 \approx 19$, $\mu = 35 + 1.28(18.8) \approx 59$.

PYQ. A mean of a normal population equals the standard error of the mean of samples of size 64. Find the probability that the mean of a sample of size 64 will be negative. Apr/May-2023 · 5M

$\mu = \frac{\sigma}{\sqrt{64}} = \frac{\sigma}{8}$. $Z = \frac{\bar{x} - \mu}{\sigma/8}$. For $\bar{x} = 0$: $Z = \frac{0 - \sigma/8}{\sigma/8} = -1$.
 $P(\bar{x} < 0) = P(Z < -1) = 0.5 - 0.3413 = 0.1587$.

2.8 Comparison of Binomial, Poisson and Normal Oct-2025 · 5M

Feature	Binomial $B(n, p)$	Poisson $P(\lambda)$	Normal $N(\mu, \sigma^2)$
Type	Discrete	Discrete	Continuous
Values	$0, 1, \dots, n$	$0, 1, 2, \dots, \infty$	$-\infty$ to ∞
Formula	${}^nC_r p^r q^{n-r}$	$\frac{e^{-\lambda} \lambda^r}{r!}$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
Parameters	n, p	λ	μ, σ
Mean	np	λ	μ
Variance	npq (< mean)	λ (= mean)	σ^2 (independent of mean)
Shape	Symmetric only if $p = \frac{1}{2}$	Right-skewed; near-symmetric for large λ	Always symmetric, bell-shaped
Used when	Fixed n , two outcomes	n large, p small (rare events)	Measurements; large n

Links: Binomial $\rightarrow$ Poisson when $n \rightarrow \infty$, $p \rightarrow 0$, $np = \lambda$. Binomial $\rightarrow$ Normal when n is large and p is not near 0 or 1 ($\mu = np$, $\sigma = \sqrt{npq}$).

Example 13. Find the probability that between 84 and 95 (inclusive) of 100 patients survive a heart operation, if the chance of survival is 0.9. Class notes

Use the normal approximation to the binomial: $\mu = np = 90$, $\sigma = \sqrt{npq} = \sqrt{9} = 3$.
 $z_1 = \frac{84-90}{3} = -2$, $z_2 = \frac{95-90}{3} = 1.67$. $P = A(2) + A(1.67) = 0.4772 + 0.4525 = 0.9297$.

2.9 Sampling Distribution and Standard Error

Population Mar/Apr-2024 · 2M

The full set of observations that we study (finite or infinite). Its size is N .

Sample

A part of the population that we examine to learn about the population. Its size is n . The process is **sampling**.

Parameter

A number that describes the **population**: μ, σ, P .

Statistic

A number that we calculate from the **sample**: $\bar{x}, s, p$. A statistic changes from sample to sample.

Random sampling

Each member of the population has an equal chance to be in the sample.

Large vs small sample

$n \geq 30$: large sample. $n < 30$: small sample.

With replacement

An element can be selected again. No. of samples = N^n .

Without replacement

An element cannot be selected again. No. of samples = ${}^N C_n$.

SAMPLING DISTRIBUTION AND STANDARD ERROR

Take all possible samples of size n . Calculate the statistic (for example $\bar{x}$) for each sample. The distribution of these values is the **sampling distribution** of the statistic.

The standard deviation of a sampling distribution is the **standard error (S.E.)** of that statistic. The S.E. measures how much the statistic changes from sample to sample.

SAMPLE MEAN, SAMPLE VARIANCE, STANDARD ERRORS

$\bar{x} = \frac{\sum x_i}{n}$; $s^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ (or $\frac{\sum (x_i - \bar{x})^2}{n-1}$ for an unbiased estimate).

Statistic	Standard error
Mean $\bar{x}$ (infinite pop. / with replacement)	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
Mean $\bar{x}$ (finite pop., without replacement)	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$
Difference of means $\bar{x}_1 - \bar{x}_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
Proportion p	$\sqrt{\frac{PQ}{n}}$
Difference of proportions	$\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}$
Sample s.d. s	$\frac{\sigma}{\sqrt{2n}}$
Difference of s.d.s	$\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}$

Mean of the sampling distribution of means: $\mu_{\bar{x}} = \mu$ (always).

CENTRAL LIMIT THEOREM

Let $\bar{x}$ be the mean of a random sample of size n from a population with mean μ and s.d. σ . For large n , the sampling distribution of $\bar{x}$ is approximately normal with mean μ and s.d. $\frac{\sigma}{\sqrt{n}}$. So $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ is approximately $N(0, 1)$.

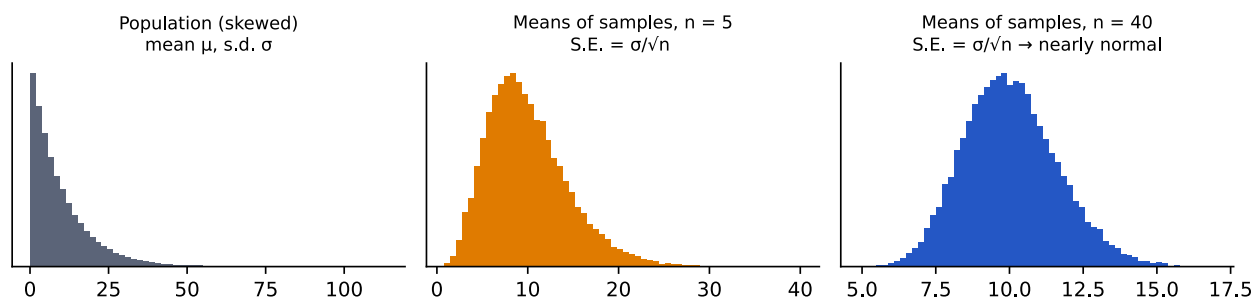


Fig 2.7 — Central limit theorem: sample means become normal as n increases, even from a skewed population.

PYQ. A population consists of the five numbers 2, 3, 6, 8, 11. Consider all possible samples of size 2 drawn **with replacement**. Find (a) the mean of the population (b) the s.d. of the population (c) the mean of the sampling distribution of means (d) the s.d. of the sampling distribution of means.

[Class notes](#)

[Nov-2024 \(size 3\)](#)

(a) $\mu = \frac{2+3+6+8+11}{5} = 6$. (b) $\sigma^2 = \frac{16+9+0+4+25}{5} = 10.8$, $\sigma = 3.286$.

(c) No. of samples = $N^n = 5^2 = 25$. The 25 sample means add to 150. $\mu_{\bar{x}} = \frac{150}{25} = 6$.

	2	3	6	8	11
2	2	2.5	4	5	6.5
3	2.5	3	4.5	5.5	7
6	4	4.5	6	7	8.5
8	5	5.5	7	8	9.5
11	6.5	7	8.5	9.5	11

(d) $\sum(\bar{x} - 6)^2 = 135$. $\sigma_{\bar{x}}^2 = \frac{135}{25} = 5.4$, $\sigma_{\bar{x}} = 2.32$.

Check by formula: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.286}{\sqrt{2}} = 2.32$ ✓.

Nov-2024 version (size 3, with replacement): $5^3 = 125$ samples; $\mu_{\bar{x}} = 6$; $\sigma_{\bar{x}} = \frac{3.286}{\sqrt{3}} = 1.897$.

6, 3.286, 6, 1.897

PYQ. A population consists of 2, 3, 5, 6, 8. Consider all samples of size 3 drawn **without replacement**. Find (a) μ (b) σ (c) mean of the sampling distribution of means (d) s.d. of the sampling distribution of means. [May-2025 · 10M](#)

(a) $\mu = \frac{24}{5} = 4.8$. (b) $\sigma^2 = \frac{\sum x^2}{5} - \mu^2 = \frac{138}{5} - 23.04 = 4.56$, $\sigma = 2.135$.

(c) ${}^5C_3 = 10$ samples. Means: 3.33, 3.67, 4.33, 4.33, 4.67, 5, 5.33, 5.33, 5.67, 6.33. Their sum is 48, so their mean = $4.8 = \mu$.

(d) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = \frac{2.135}{\sqrt{3}} \sqrt{\frac{2}{4}} = 0.872$. **4.8, 2.135, 4.8, 0.872**

PYQ. A population consists of 3, 6, 9, 15, 27. Consider all samples of size 2 drawn without replacement. Find (a)–(d) as above. Oct-2023 · 10M

$$\mu = 12; \sigma^2 = \frac{81+36+9+9+225}{5} = 72, \sigma = 8.485; {}^5C_2 = 10 \text{ samples; } \mu_{\bar{x}} = 12;$$

$$\sigma_{\bar{x}} = \frac{8.485}{\sqrt{2}} \sqrt{\frac{3}{4}} = 5.196. \quad \boxed{12, 8.485, 12, 5.196}$$

PYQ. Populations 5, 10, 14, 15, 18, 24 (Oct-2021) and 4, 8, 12, 16, 20, 24 (Nov-2020): all samples of size 2 without replacement. Oct/Nov-2021 · 15M Nov-2020 · 8M

Population	μ	σ^2 (σ)	No. of samples	$\mu_{\bar{x}}$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{2}} \sqrt{\frac{4}{5}}$
5,10,14,15,18,24	14.33	35.56 (5.963)	15	14.33	3.771
4,8,12,16,20,24	14	46.67 (6.831)	15	14	4.320 (variance 18.67)

Class notes use 5, 10, 14, 18, 13, 24: $\mu = 14, \sigma = 5.97, \sigma_{\bar{x}} = 3.78$.

PYQ. If the mean and s.d. of a sampling distribution are 100 and 15, find the standard error for a sample of size 25. Oct-2025 · 5M Class notes

$$\text{S.E.} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{25}} = 3.$$

Example 14. (a) A sample of 100 is taken from a population with $\mu = 76, \sigma^2 = 256$. Find $P(75 < \bar{x} < 78)$. (b) A normal population has mean 0.1 and s.d. 2.1. Find the probability that the mean of a sample of 900 is negative. Class notes

(a) $\text{S.E.} = \frac{16}{10} = 1.6. z_1 = \frac{75-76}{1.6} = -0.63, z_2 = \frac{78-76}{1.6} = 1.25. P = A(0.63) + A(1.25) = 0.2357 + 0.3944 = 0.6301$

(b) $\text{S.E.} = \frac{2.1}{30} = 0.07. z = \frac{0-0.1}{0.07} = -1.43. P = 0.5 - A(1.43) = 0.5 - 0.4236 = 0.0764.$

2.10 Quick Revision Sheet

Discrete RV

$$\sum p_i = 1; \mu = \sum x_i p_i; \sigma^2 = \sum x_i^2 p_i - \mu^2$$

Continuous RV

$$\int f = 1; \mu = \int x f; \sigma^2 = \int x^2 f - \mu^2; \text{median: } \int_{-\infty}^M f = \frac{1}{2}$$

Binomial

$${}^n C_r p^r q^{n-r}; \mu = np; \sigma^2 = npq$$

Poisson

$$\frac{e^{-\lambda} \lambda^r}{r!}; \lambda = np; \mu = \sigma^2 = \lambda$$

Normal

$$Z = \frac{X - \mu}{\sigma}; 68.26 / 95.44 / 99.73 \%$$

Sampling

$$\mu_{\bar{x}} = \mu; \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \left(\times \sqrt{\frac{N-n}{N-1}} \text{ without repl.} \right)$$

Key z values

$$z_{0.05}: 1.645 \text{ (one tail)} \cdot 1.96 \text{ (two tails)} \cdot z_{0.01}: 2.33 \cdot 2.58 \cdot 10\%: 1.28$$

Expectation

$$E(aX + b) = aE(X) + b; \text{Var}(aX + b) = a^2 \text{Var}(X)$$

Normal table — area $A(z)$ from 0 to z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

★ Assignment — Unit 2 (5 questions)

1. Define a random variable. Distinguish between discrete and continuous random variables with one example each. [5M]
2. A continuous RV has PDF $f(x) = kx^2$ for $0 \leq x \leq 3$, and 0 elsewhere. Find k , $P(1 \leq X \leq 2)$, the mean and the variance. [10M]
3. Derive the mean and variance of the Poisson distribution. Six coins are tossed 6400 times. Using the Poisson distribution, find the approximate probability of getting six heads r times. [10M]
4. In a normal distribution, 17% of items are below 30 and 17% are above 60. Find the mean and standard deviation. [5M]
5. A population consists of 1, 5, 6, 8. Write all samples of size 2 drawn with replacement. Find the mean and standard error of the sampling distribution of means, and verify with the formula. [10M]

Answers for self-check: Q2: $k = \frac{1}{9}$, $P = \frac{7}{27}$, $\mu = 2.25$, $\sigma^2 = 0.3375$. Q3: $p = \frac{1}{64}$, $\lambda = 100$, $P(r) = \frac{e^{-100} 100^r}{r!}$. Q4: $z = \pm 0.95$, $\mu = 45$, $\sigma = 15.8$. Q5: 16 samples, $\mu_{\bar{x}} = 5$, $\sigma = 2.55$, $\sigma_{\bar{x}} = \frac{2.55}{\sqrt{2}} = 1.80$.