

Unit 3 — Testing of Hypothesis I

Estimation (point & interval) · Null & alternative hypothesis · Type I & II errors · One / two-tailed tests · Critical region · Large sample tests: one mean, two means, one proportion, two proportions

Exam slot: Part-B Q6 / Q7 (10 M)

Part-A: 1e, 1f (2 + 3 M)

Written in ASD-STE100 style

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ABOUT THIS UNIT

Your class notes for Unit 3 were not available. These notes follow the **same teaching pattern as your Unit 4 notes**: the 6-step test ($H_0 \rightarrow H_1 \rightarrow$ level of significance $\rightarrow$ test statistic $\rightarrow$ tabulated value $\rightarrow$ conclusion). The only change is the test statistic: Unit 3 uses **Z** (large samples, $n \geq 30$). Unit 4 uses **t**, **F**, **χ^2** (small samples).

PYQ TREND (2020 – 2026): WHAT THE EXAM ASKS FROM THIS UNIT

- **Single mean Z-test + confidence interval** (ocean depth, sample of 400, 40 engineers): every paper.
- **Difference of two means** (bulbs, samples of 1000 and 2000, men vs women weights).
- **Single / two proportions** (wrong lane 30%, allopathy vs homeopathy, vending machines, smokers).
- **Short questions**: critical region & LOS, Type I/II errors, null/alternative hypothesis, one vs two tailed, good estimator, sample size, maximum error.

3.1 Basic terms of estimation

3.2 Point estimation & good estimator

3.3 Interval estimation, max error, sample size

3.4 Statistical hypothesis: H_0 and H_1

3.5 Type I and Type II errors

3.6 LOS, critical region, one / two-tailed

3.7 Procedure for testing a hypothesis

3.8 Test for single mean

3.9 Test for difference of means

3.10 Test for single proportion

3.11 Test for difference of proportions

3.12 Revision sheet

★ Assignment (5 questions)

3.1 Basic Terms

Parameter

A number that describes the population: μ , σ^2 , P . Usually it is **unknown**.

Statistic

A number that we calculate from a sample: $\bar{x}$, s^2 , p . We use it to estimate the parameter.

Estimator

The **rule (formula)** that gives an estimate.
Example: $\bar{x} = \frac{\sum x_i}{n}$ is an estimator of μ .

Estimate

The **value** that the estimator gives for one sample. Example: $\bar{x} = 62$ kg.

ESTIMATION — IMPORTANCE AND TYPES APR/MAY-2026 · 3M

Estimation is the method to find the unknown value of a population parameter from sample data.

Importance: We cannot measure every member of a large population (cost, time, destructive tests). Estimation gives the value of the parameter from a small sample. It also tells us how accurate that value is.

Types: (1) **Point estimation** — one single value. (2) **Interval estimation** — a range of values with a stated confidence.

3.2 Point Estimation**POINT ESTIMATION** MAR/APR-2024 · 2M OCT-2025 · 5M

A **point estimate** of a parameter is a **single number** calculated from the sample. We use this number as the value of the parameter.

Example: A sample of 50 students has mean weight 62 kg. The point estimate of the population mean is $\hat{\mu} = \bar{x} = 62$ kg.

Common point estimates: $\hat{\mu} = \bar{x}$, $\hat{\sigma}^2 = S^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$, $\hat{P} = p = \frac{x}{n}$.

PROPERTIES OF A GOOD ESTIMATOR APR/MAY-2023 · 3M

1. **Unbiasedness:** $E(\hat{\theta}) = \theta$. The mean of the estimator equals the parameter. May-2025 · 2M

Examples: $E(\bar{x}) = \mu$, so $\bar{x}$ is unbiased for μ . $S^2 = \frac{\sum(x - \bar{x})^2}{n-1}$ is unbiased for σ^2 ; $s^2 = \frac{\sum(x - \bar{x})^2}{n}$ is biased.

2. **Consistency:** As $n \rightarrow \infty$, $\hat{\theta} \rightarrow \theta$ (the estimate becomes closer to the true value).

3. **Efficiency:** Of two unbiased estimators, the one with the smaller variance is more efficient. ($\bar{x}$ is more efficient than the median.)

4. **Sufficiency:** The estimator uses all the information in the sample about the parameter.

3.3 Interval Estimation**INTERVAL ESTIMATION AND CONFIDENCE INTERVAL**

An **interval estimate** is a range (a, b) in which the parameter lies, with a stated probability. The probability is the **confidence level** $(1 - \alpha)$. The interval is the **confidence interval**; a and b are the **confidence limits**.

Example: "The mean weight is between 60.6 kg and 63.4 kg, with 95% confidence."

So the interval $\bar{x} \pm 1.96 \sigma/\sqrt{n}$ contains μ in 95% of samples

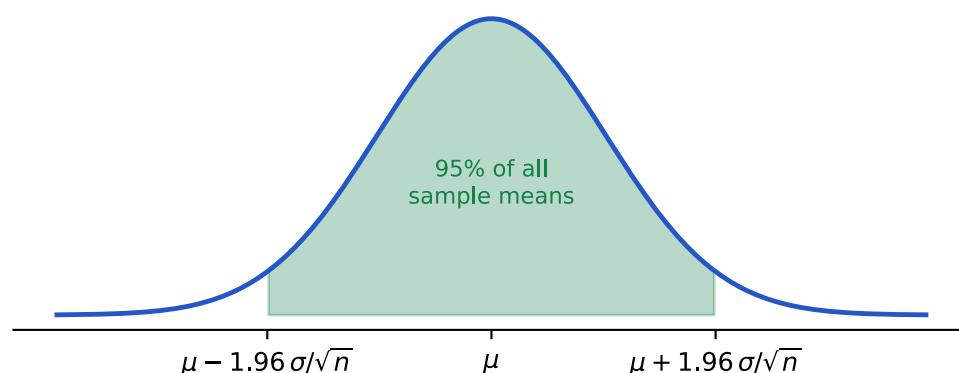


Fig 3.1 — Why $\bar{x} \pm 1.96 \sigma/\sqrt{n}$ is a 95% confidence interval.

CONFIDENCE INTERVALS (LARGE SAMPLES, $N \geq 30$)

Parameter	Confidence interval
Mean μ	$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ (use s if σ is not known)
Proportion P	$p \pm z_{\alpha/2} \sqrt{\frac{pq}{n}}$
Difference of means	$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
Difference of proportions	$(p_1 - p_2) \pm z_{\alpha/2} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

Confidence	90%	95%	98%	99%
$z_{\alpha/2}$	1.645	1.96	2.33	2.58

Maximum error of estimate: $E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$. **Sample size:** $n = \left(\frac{z_{\alpha/2} \sigma}{E} \right)^2$ (round up).

PYQ. Define point and interval estimation. Explain with examples. Oct-2025 · 5M

Use the definitions in 3.2 and 3.3. Example: 50 students, $\bar{x} = 62$ kg, $\sigma = 5$ kg.

Point estimate of μ : 62 kg. 95% interval estimate: $62 \pm 1.96 \cdot \frac{5}{\sqrt{50}} = 62 \pm 1.386 = (60.61, 63.39)$ kg.

PYQ. To estimate the average time visitors take to move from one building to another, a sample of size n is used. Given $\sigma = 1.40$ minutes, how large should the sample be to be 99% confident that the error E is at most 0.25? May-2025 · 3M Nov-2024 · 3M

$$n = \left(\frac{z_{\alpha/2} \sigma}{E} \right)^2 = \left(\frac{2.58 \times 1.40}{0.25} \right)^2 = (14.448)^2 = 208.7. \text{ Round up. } \mathbf{n = 209}$$

PYQ. A random sample of 400 items has mean 82 and s.d. 18. Find the maximum error of estimate at 95% confidence. Nov/Dec-2022 · 3M

$$E = 1.96 \times \frac{18}{\sqrt{400}} = 1.96 \times 0.9 = 1.764. \mathbf{E = 1.764}$$

PYQ. The efficiency expert of a computer company tested 40 engineers. The mean time to assemble a component is 12.73 min and s.d. 2.06 min. (a) If $\bar{x} = 12.73$ is used as a point estimate, find the maximum error with 99% confidence. (b) Construct a 98% confidence interval. (c) With what confidence can we assert that the sample mean does not differ from the true mean by more than 30 seconds? Mar/Apr-2024 · 10M Oct-2023 · 3M (part c)

$$\text{S.E.} = \frac{s}{\sqrt{n}} = \frac{2.06}{\sqrt{40}} = 0.3257.$$

Step 1 (a) $E = 2.58 \times 0.3257 = 0.840$ min.

Step 2 (b) $12.73 \pm 2.33 \times 0.3257 = 12.73 \pm 0.759 = (11.97, 13.49)$ min.

Step 3 (c) 30 seconds = 0.5 min. $z = \frac{E}{\text{S.E.}} = \frac{0.5}{0.3257} = 1.535$. Confidence = $2A(1.535) = 2(0.4376) = 0.875$.

(a) 0.84 min (b) (11.97, 13.49) (c) $\approx 87.5\%$

PYQ. A random sample of 100 is taken from a population with $\sigma = 5.1$. The sample mean is 21.6. Construct a 95% confidence interval for μ . Mar/Apr-2022 · 6M

$$21.6 \pm 1.96 \times \frac{5.1}{\sqrt{100}} = 21.6 \pm 1.0 = (20.6, 22.6).$$

Same method: $n = 400$, s.d. 10, mean 60 $\rightarrow$ (59.02, 60.98) Oct/Nov-2021; $n = 200$, mean 50, s.d. 9 $\rightarrow$ (48.75, 51.25) Mar/Apr-2022.

PYQ. Construct a 95% confidence interval for the true proportion of computer literates if 47 out of 150 persons from rural areas are computer literates. Apr/May-2023 · 5M Jul-2021 · 7M

$$p = \frac{47}{150} = 0.3133, q = 0.6867. \text{ S.E. } = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.3133 \times 0.6867}{150}} = 0.0379.$$

$$\text{CI} = 0.3133 \pm 1.96(0.0379) = 0.3133 \pm 0.0742 = (0.239, 0.388). \quad \text{23.9\% to 38.8\%}$$

3.4 Statistical Hypothesis

STATISTICAL HYPOTHESIS AND TEST OF HYPOTHESIS JUL-2021 · 7M

A **statistical hypothesis** is an assumption (statement) about a population parameter. It may be true or false. *Example:* "The mean life of the bulbs is 1000 hours."

A **test of hypothesis** is a procedure that uses sample data to decide whether to accept or reject the hypothesis.

Null hypothesis H_0 Nov/Dec-2022 · 2M Oct-2023 · 2M

The hypothesis of **no difference**. We test it for possible rejection. It always contains "=".

Example: $H_0 : \mu = 60$ (the population mean is 60 kg). $H_0 : \mu_1 = \mu_2$.

Alternative hypothesis H_1

The hypothesis that we accept when we reject H_0 . It is the **opposite** of H_0 .

$H_1 : \mu \neq 60$ (two-tailed) $H_1 : \mu > 60$ (right-tailed) $H_1 : \mu < 60$ (left-tailed)

HOW TO SELECT H_1 FROM THE WORDS OF THE QUESTION

Words in question	H_1	Test
"is there a difference", "same", "equal", "regarded as drawn from", "test whether mean is 60"	$\neq$	Two-tailed
"superior", "better", "more", "greater", "increase", "longer", "heavier"	$>$	Right-tailed
"less", "at least" claim being doubted, "decrease", "inferior"	$<$	Left-tailed

3.5 Type I and Type II Errors Oct-2025 · 2M Apr/May-2023 · 2M

Decision $\downarrow$ / Truth $\rightarrow$	H_0 is true	H_0 is false
Accept H_0	Correct decision	Type II error (β)
Reject H_0	Type I error (α)	Correct decision

Type I error

We **reject** H_0 when H_0 is true.

$P(\text{Type I error}) = \alpha = \text{level of significance}$. Also called **producer's risk**.

Example: A good batch of bulbs is rejected.

Type II error

We **accept** H_0 when H_0 is false.

$P(\text{Type II error}) = \beta$. Also called **consumer's risk**. $1 - \beta$ is the **power** of the test.

Example: A bad batch of bulbs is accepted.

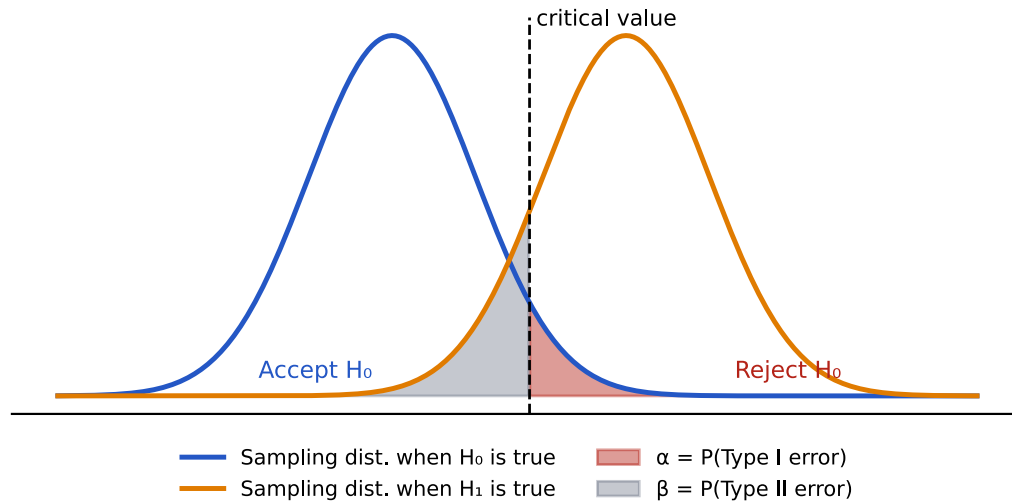


Fig 3.2 — α and β . If we move the critical value to decrease α , β increases (for a fixed n).

3.6 Level of Significance, Critical Region, One & Two-tailed Tests

LEVEL OF SIGNIFICANCE (LOS) AND CRITICAL REGION

APR/MAY-2026 · 2M

NOV-2024 · 2M

OCT-2025 · 5M

Level of significance α : the maximum probability of a Type I error that we accept. Usually $\alpha = 5\%$ (0.05) or 1% (0.01). At $\alpha = 5\%$ we are 95% confident in our decision.

Critical region (rejection region): the part of the sampling distribution in which we **reject** H_0 . Its area is α . The other part is the **acceptance region**.

Critical value z_α : the value that separates the two regions.

Importance: The critical region gives a fixed, objective rule. If the calculated value of the test statistic falls inside it, we reject H_0 . So two persons with the same data get the same decision.

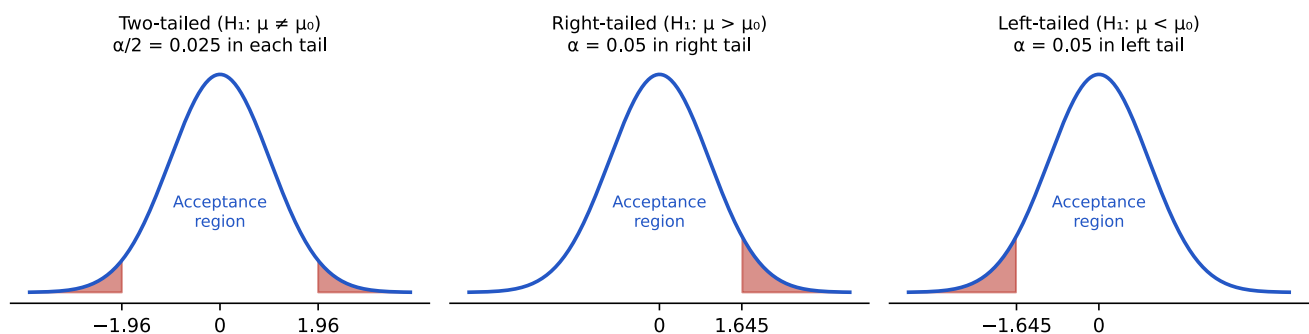


Fig 3.3 — Critical regions (red) at $\alpha = 5\%$ for the three types of H_1 .

CRITICAL VALUES OF Z (LEARN THIS TABLE)

Level of significance	1%	2%	5%	10%
Two-tailed test $ z_\alpha $	2.58	2.33	1.96	1.645
Right-tailed z_α	2.33	2.055	1.645	1.28
Left-tailed z_α	-2.33	-2.055	-1.645	-1.28

PYQ. Explain the difference between one-tailed and two-tailed tests. Oct-2025 · 3M

One-tailed test	Two-tailed test
H_1 has $>$ or $<$.	H_1 has $\neq$.
Critical region is in one tail; area α .	Critical region is in both tails; area $\alpha/2$ each.
At 5%: $z_\alpha = 1.645$.	At 5%: $z_\alpha = 1.96$.
Example: new bulbs are better ($\mu > 1000$).	Example: mean life is 1000 ($\mu \neq 1000$).

3.7 Procedure for Testing a HypothesisApr/May-2026 · 3MNov/Dec-2022 · 5MOct/Nov-2021 · 7M

Six steps of a test of hypothesis (use this order in every answer)

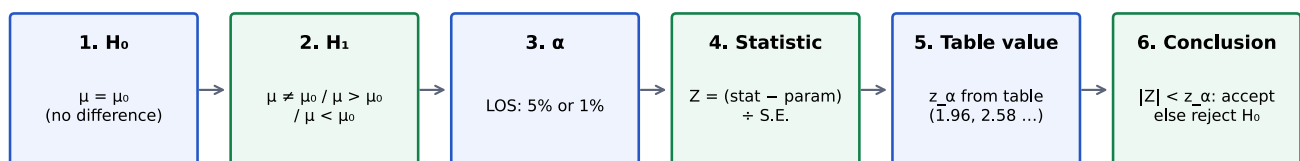


Fig 3.4 — The 6 steps. Your class notes use the same steps in Unit 4.

- Step 1** **Null hypothesis H_0 :** Write the assumption of no difference, e.g. $H_0 : \mu = \mu_0$.
- Step 2** **Alternative hypothesis H_1 :** Write $\mu \neq \mu_0$ or $\mu > \mu_0$ or $\mu < \mu_0$. This tells you one- or two-tailed.
- Step 3** **Level of significance:** Select α (5% if the question does not give it).
- Step 4** **Test statistic:** Calculate $Z = \frac{t - E(t)}{\text{S.E.}(t)}$, where t is the sample statistic.
- Step 5** **Tabulated (critical) value:** Read z_α for the given α and tail.
- Step 6** **Conclusion:** If $|Z| < z_\alpha$, accept H_0 . Otherwise reject H_0 . Write the decision in the words of the question.

CAUTION — EXAM MARKS

Write all 6 step headings in every answer. Examiners give marks for each step. Always end with one sentence in plain words, for example "So the bulbs of company A are not superior."

3.8 Large Sample Test for Single Mean

TEST STATISTIC ($N \geq 30$)

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad (\text{use } s \text{ for } \sigma \text{ when } \sigma \text{ is not known})$$

Confidence limits for μ : $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$.

PYQ. A sample of 50 students has a mean weight of 62 kg. The population s.d. is 5 kg. Formulate H_0 and H_1 to test if the population mean is 60 kg at 5% level. **Oct-2025 · 5M**

Step 1 $H_0 : \mu = 60 \text{ kg.}$

Step 2 $H_1 : \mu \neq 60 \text{ kg (two-tailed).}$

Step 3 $\alpha = 0.05.$

Step 4 $Z = \frac{62 - 60}{5/\sqrt{50}} = \frac{2}{0.7071} = 2.83.$

Step 5 Tabulated $z_{0.05} = 1.96.$

Step 6 $|Z| = 2.83 > 1.96.$ Reject H_0 . The population mean is not 60 kg.

PYQ. A sample of 400 items is taken from a population whose s.d. is 10. The mean of the sample is 40. Test whether the sample has come from a population with mean 38. Also calculate the 95% confidence interval for the population mean. **May-2025 · 10M**

Step 1 $H_0 : \mu = 38.$

Step 2 $H_1 : \mu \neq 38 \text{ (two-tailed).}$

Step 3 $\alpha = 0.05.$

Step 4 $Z = \frac{40 - 38}{10/\sqrt{400}} = \frac{2}{0.5} = 4.$

Step 5 $z_{0.05} = 1.96.$

Step 6 $4 > 1.96.$ Reject H_0 . The sample is not from a population with mean 38.

95% CI: $40 \pm 1.96(0.5) = 40 \pm 0.98 = (39.02, 40.98).$ **Z = 4, reject; CI (39.02, 40.98)**

PYQ. An oceanographer wants to check whether the depth of the ocean in a region is 57.4 fathoms, as previously recorded. At $\alpha = 0.05$, what can he conclude if soundings at 40 random locations gave a mean of 59.1 fathoms with s.d. 5.2? Also calculate the 95% confidence interval. **Mar/Apr-2024 · 10M**

Step 1 $H_0 : \mu = 57.4.$

Step 2 $H_1 : \mu \neq 57.4.$

Step 3 $\alpha = 0.05.$

Step 4 S.E. = $\frac{5.2}{\sqrt{40}} = 0.822.$ $Z = \frac{59.1 - 57.4}{0.822} = 2.07.$

Step 5 $z_{0.05} = 1.96.$

Step 6 $2.07 > 1.96.$ Reject H_0 . The depth is not 57.4 fathoms.

95% CI: $59.1 \pm 1.96(0.822) = (57.49, 60.71)$ fathoms.

PYQ. The average depth of ocean in a region is recorded as 67.4 fathoms. Is there reason to believe this at the 0.01 level, if readings at 40 locations showed a mean of 69.3 with s.d. 5.4? **Oct-2023 · 10M**

$$H_0 : \mu = 67.4; H_1 : \mu \neq 67.4; \alpha = 0.01. Z = \frac{69.3 - 67.4}{5.4/\sqrt{40}} = \frac{1.9}{0.8538} = 2.23.$$

$z_{0.01} = 2.58. 2.23 < 2.58.$ Accept H_0 . The recorded depth 67.4 is acceptable.

PYQ. It is claimed that a random sample of 49 tires has a mean life of 15200 km and is drawn from a population with mean 15150 km and s.d. 1200 km. Test the claim at (i) 0.01 (ii) 0.05. **Nov/Dec-2022 · 5M**

$$H_0 : \mu = 15150; H_1 : \mu \neq 15150. Z = \frac{15200 - 15150}{1200/7} = \frac{50}{171.43} = 0.29.$$

$0.29 < 2.58$ and $0.29 < 1.96.$ Accept H_0 at both levels. The claim is valid.

PYQ. A trucking firm suspects the claim that the average life of certain tires is at least 28,000 miles. 40 tires gave a mean life of 27,463 miles with s.d. 1,348 miles. What can it conclude if the probability of a Type I error is at most 0.01? **Mar/Apr-2022 · 8M**

Step 1 $H_0 : \mu = 28000.$

Step 2 $H_1 : \mu < 28000$ (left-tailed; the claim "at least" is doubted).

Step 3 $\alpha = 0.01.$

Step 4 $Z = \frac{27463 - 28000}{1348/\sqrt{40}} = \frac{-537}{213.1} = -2.52.$

Step 5 Left-tailed $z_{0.01} = -2.33.$

Step 6 $-2.52 < -2.33:$ Z is in the critical region. Reject H_0 . The claim is not true.

3.9 Large Sample Test for Difference of Means **Nov-2020 · 6M (explain)**

TEST STATISTIC

$$H_0 : \mu_1 = \mu_2.$$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

- If σ_1, σ_2 are not known, use s_1, s_2 .

- If both samples come from **one population with s.d. σ** : $Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}.$

PYQ. The means of two large samples of sizes 1000 and 2000 are 67.5 and 68.0 inches. Can the samples be regarded as drawn from the same population of s.d. 2.5 inches? **May-2025 · 10M**

Mar/Apr-2022 · 9M

Step 1 $H_0 : \mu_1 = \mu_2$ (same population).

Step 2 $H_1 : \mu_1 \neq \mu_2$.

Step 3 $\alpha = 0.05$.

Step 4 $Z = \frac{67.5 - 68.0}{2.5 \sqrt{\frac{1}{1000} + \frac{1}{2000}}} = \frac{-0.5}{2.5 \times 0.03873} = -5.16$.

Step 5 $z_{0.05} = 1.96$.

Step 6 $|Z| = 5.16 > 1.96$. Reject H_0 . The samples are not from the same population.

PYQ. Test the significance of the difference between two sample means: $n_1 = 40$, $\bar{x}_1 = 78$, $s_1^2 = 40$; $n_2 = 35$, $\bar{x}_2 = 72$, $s_2^2 = 35$ at 5% level. **Oct-2025 · 5M**

$H_0 : \mu_1 = \mu_2$; $H_1 : \mu_1 \neq \mu_2$. $Z = \frac{78 - 72}{\sqrt{\frac{40}{40} + \frac{35}{35}}} = \frac{6}{\sqrt{2}} = 4.24 > 1.96$. Reject H_0 . The difference is significant.

PYQ. A company claims that its bulbs are superior to those of its main competitor. A sample of 40 bulbs has mean life 647 h with s.d. 27 h. A sample of 40 bulbs of the competitor has mean life 638 h with s.d. 31 h. Does this substantiate the claim at the 0.05 level? **Nov-2024 · 10M**

Step 1 $H_0 : \mu_1 = \mu_2$.

Step 2 $H_1 : \mu_1 > \mu_2$ ("superior" $\rightarrow$ right-tailed).

Step 3 $\alpha = 0.05$.

Step 4 $Z = \frac{647 - 638}{\sqrt{\frac{27^2}{40} + \frac{31^2}{40}}} = \frac{9}{\sqrt{18.225 + 24.025}} = \frac{9}{6.5} = 1.38$.

Step 5 Right-tailed $z_{0.05} = 1.645$.

Step 6 $1.38 < 1.645$. Accept H_0 . The data do not support the claim of superiority.

PYQ. A sample of 100 bulbs by manufacturer A has mean life 1190 h, s.d. 90 h. A sample of 75 bulbs by B has mean life 1230 h, s.d. 120 h. Is there a difference between the mean lives at the 0.05 level? **Apr/May-2023 · 5M**

$H_0 : \mu_1 = \mu_2$; $H_1 : \mu_1 \neq \mu_2$. $Z = \frac{1190 - 1230}{\sqrt{\frac{8100}{100} + \frac{14400}{75}}} = \frac{-40}{\sqrt{81 + 192}} = \frac{-40}{16.52} = -2.42$.

$|Z| = 2.42 > 1.96$. Reject H_0 . The mean lives are different.

PYQ. 6400 men have mean weight 67.85 kg, s.d. 2.56 kg. 1600 women have mean 68.55 kg, s.d. 2.52 kg. Do the data indicate that men are, on average, heavier than women? **Nov/Dec-2022 · 5M**

$$H_0 : \mu_1 = \mu_2; H_1 : \mu_1 > \mu_2 \text{ (right-tailed)}. Z = \frac{67.85 - 68.55}{\sqrt{\frac{2.56^2}{6400} + \frac{2.52^2}{1600}}} = \frac{-0.70}{0.0707} = -9.91.$$

Right-tailed $z_{0.05} = 1.645$. Z is negative, so it is not in the right-tail critical region. Accept H_0 : the data do **not** show that men are heavier. (In fact $|Z|$ is very large, so women are significantly heavier in this sample.)

3.10 Large Sample Test for Single Proportion

TEST STATISTIC

$p = \frac{x}{n}$ = sample proportion, P = population proportion given in H_0 , $Q = 1 - P$.

$$Z = \frac{p - P}{\sqrt{PQ/n}} \quad \text{Confidence limits for } P : p \pm z_{\alpha/2} \sqrt{\frac{pq}{n}}$$

Note: in the test, the S.E. uses P from H_0 . In the confidence interval, the S.E. uses p from the sample.

PYQ. In a random sample of 1200 cars making a right turn at a junction, 314 drove into the wrong lane. Test whether actually 30% of all drivers make this mistake. Use (a) 0.05 (b) 0.01 level. Comment on the results. **Nov-2024 · 10M**

Step 1 $H_0 : P = 0.30$.

Step 2 $H_1 : P \neq 0.30$.

Step 3 (a) $\alpha = 0.05$, (b) $\alpha = 0.01$.

Step 4 $p = \frac{314}{1200} = 0.2617$. $Z = \frac{0.2617 - 0.30}{\sqrt{0.3 \times 0.7/1200}} = \frac{-0.0383}{0.01323} = -2.90$.

Step 5 (a) 1.96 (b) 2.58.

Step 6 $|Z| = 2.90$ is greater than both. Reject H_0 at both levels. Comment: the true proportion is not 30%; it is less than 30%. The result is strong, because it is significant even at 1%.

PYQ. A manufacturer claims that only 4% of his products are defective. A random sample of 500 had 100 defective. Test the hypothesis at the 0.05 level. **Mar/Apr-2022 · 6M**

$$H_0 : P = 0.04; H_1 : P > 0.04 \text{ (right-tailed)}. p = \frac{100}{500} = 0.2.$$

$$Z = \frac{0.2 - 0.04}{\sqrt{0.04 \times 0.96/500}} = \frac{0.16}{0.00876} = 18.26 > 1.645. \text{ Reject } H_0. \text{ The claim is false.}$$

3.11 Large Sample Test for Difference of Proportions

TEST STATISTIC

$$p_1 = \frac{x_1}{n_1}, p_2 = \frac{x_2}{n_2}. \text{ Pooled proportion } P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}, Q = 1 - P.$$

$$Z = \frac{p_1 - p_2}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

PYQ. If 57 out of 150 patients with a disease are cured by allopathy and 33 out of 100 by homeopathy, is there reason to believe that allopathy is better than homeopathy at the 0.05 level?

Oct-2023 · 10M

Oct/Nov-2021 · 8M

Step 1 $H_0 : P_1 = P_2.$

Step 2 $H_1 : P_1 > P_2$ ("better" → right-tailed).

Step 3 $\alpha = 0.05.$

Step 4 $p_1 = \frac{57}{150} = 0.38, p_2 = \frac{33}{100} = 0.33. P = \frac{57+33}{250} = 0.36, Q = 0.64.$
 $S.E. = \sqrt{0.36 \times 0.64 \left(\frac{1}{150} + \frac{1}{100} \right)} = \sqrt{0.2304 \times 0.01667} = 0.0620. Z = \frac{0.05}{0.062} = 0.81.$

Step 5 Right-tailed $z_{0.05} = 1.645.$

Step 6 $0.81 < 1.645.$ Accept H_0 . There is no reason to believe allopathy is better.

PYQ. The owner of a machine shop tests two snack vending machines 250 times each. The first fails 13 times and the second fails 7 times. Test at 0.05 whether the difference between the sample proportions is significant.

Apr/May-2023 · 5M

Jul-2021 · 8M

$$p_1 = \frac{13}{250} = 0.052, p_2 = \frac{7}{250} = 0.028. P = \frac{20}{500} = 0.04, Q = 0.96.$$

$$Z = \frac{0.052 - 0.028}{\sqrt{0.04 \times 0.96 \left(\frac{2}{250} \right)}} = \frac{0.024}{0.01753} = 1.37 < 1.96. \text{ Accept } H_0. \text{ The difference is not significant.}$$

PYQ. In a city, 125 men in a sample of 500 were smokers. In another city, 375 in a sample of 1000 were smokers. Does this indicate a greater proportion of smokers in the second city?

Apr/May-2023 · 5M

$$H_0 : P_1 = P_2; H_1 : P_1 < P_2 \text{ (left-tailed). } p_1 = 0.25, p_2 = 0.375. P = \frac{500}{1500} = \frac{1}{3}, Q = \frac{2}{3}.$$

$$Z = \frac{0.25 - 0.375}{\sqrt{\frac{1}{3} \cdot \frac{2}{3} \left(\frac{1}{500} + \frac{1}{1000} \right)}} = \frac{-0.125}{0.0258} = -4.84 < -1.645. \text{ Reject } H_0. \text{ Yes, the second city has a greater proportion of smokers.}$$

PYQ. In city A, 20% of a random sample of 900 school boys had a slight physical defect. In city B, 18.5% of 1600 boys had the same defect. Is the difference between the populations significant?

Nov/Dec-2022 · 5M

$$P = \frac{900(0.2) + 1600(0.185)}{2500} = \frac{180 + 296}{2500} = 0.1904, Q = 0.8096.$$

$$Z = \frac{0.2 - 0.185}{\sqrt{0.1904 \times 0.8096 \left(\frac{1}{900} + \frac{1}{1600}\right)}} = \frac{0.015}{0.01636} = 0.92 < 1.96. \text{ Accept } H_0. \text{ The difference is not significant.}$$

PYQ. A company surveys a new plan. In one group of 500 workers, 62% favoured the plan. In another group of 400 workers, 41% were against it. Is there a significant difference between the two groups at the 2% level? Oct/Nov-2021 · 8M

Group 2 favour: $100\% - 41\% = 59\%$. $p_1 = 0.62, p_2 = 0.59. P = \frac{310+236}{900} = 0.6067, Q = 0.3933.$

$$Z = \frac{0.03}{\sqrt{0.6067 \times 0.3933 \left(\frac{1}{500} + \frac{1}{400}\right)}} = \frac{0.03}{0.0328} = 0.92. \text{ At 2\% (two-tailed) } z = 2.33. \text{ Accept } H_0. \text{ No significant difference.}$$

PYQ. 400 men and 600 women were asked about a flyover. 200 men and 325 women were in favour. Test whether the proportions of men and women in favour are the same at 5%. Nov-2020 · 9M

$$p_1 = 0.5, p_2 = 0.5417. P = \frac{525}{1000} = 0.525. Z = \frac{-0.0417}{\sqrt{0.525 \times 0.475 \left(\frac{1}{400} + \frac{1}{600}\right)}} = \frac{-0.0417}{0.0322} = -1.29. |Z| < 1.96$$

. Accept H_0 .

3.12 Quick Revision Sheet

Test	H_0	Test statistic Z
Single mean	$\mu = \mu_0$	$\frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
Two means	$\mu_1 = \mu_2$	$\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$
Single proportion	$P = P_0$	$\frac{p - P_0}{\sqrt{P_0 Q_0/n}}$
Two proportions	$P_1 = P_2$	$\frac{p_1 - p_2}{\sqrt{PQ(1/n_1 + 1/n_2)}}, P = \frac{x_1 + x_2}{n_1 + n_2}$

Estimation

CI: statistic $\pm z_{\alpha/2} \times \text{S.E.}$

$$E = z_{\alpha/2} \sigma / \sqrt{n}; n = (z_{\alpha/2} \sigma / E)^2$$

Critical values

Two-tailed: 1.96 (5%), 2.58 (1%), 2.33 (2%)

One-tailed: 1.645 (5%), 2.33 (1%)

Errors

Type I: reject true H_0 (α)

Type II: accept false H_0 (β)

Decision

$|Z| < z_{\alpha} \rightarrow \text{accept } H_0$

$|Z| \geq z_{\alpha} \rightarrow \text{reject } H_0$

★ Assignment — Unit 3 (5 questions)

1. Define (i) null hypothesis (ii) alternative hypothesis (iii) Type I and Type II errors (iv) critical region. [5M]
2. A sample of 900 members has a mean of 3.4 cm and s.d. 2.61 cm. Is the sample from a large population of mean 3.25 cm? Also find the 95% confidence limits for the population mean. [10M]
3. The mean yield of wheat from district A was 210 kg with s.d. 10 kg per acre from 100 acres. District B gave mean 220 kg with s.d. 12 kg from 150 acres. Test whether there is a significant difference at 1% level. [5M]
4. A die is thrown 9000 times and a 3 or 4 appears 3240 times. Show that the die cannot be regarded as unbiased. [5M]
5. In a sample of 600 men from city A, 450 are smokers. In a sample of 900 from city B, 450 are smokers. Do the data indicate that the cities are significantly different in the habit of smoking?

Answers for self-check: Q2: $Z = \frac{0.15}{0.087} = 1.72 < 1.96$, accept; CI (3.23, 3.57). Q3: $Z = \frac{-10}{1.4} = -7.1$, reject. Q4: [10M]
 $P = \frac{1}{3}$, $p = 0.36$, $Z = \frac{0.0267}{0.00497} = 5.37 > 1.96$, reject. Q5: $p_1 = 0.75$, $p_2 = 0.5$, $P = 0.6$, $Z = \frac{0.25}{0.0258} = 9.68$, reject.