

Unit 4 — Testing of Hypothesis II

Degrees of freedom · Student t-distribution · F-distribution · χ^2 -distribution · Small sample tests: single mean, difference of means, paired t · F-test for equality of variances · χ^2 goodness of fit & independence

Exam slot: Part-B Q8 / Q9 (10 M)

Part-A: 1g, 1h (2 + 3 M)

Written in ASD-STE100 style

PYQ TREND (2020 – 2026): WHAT THE EXAM ASKS FROM THIS UNIT

- **Two horses A and B (t-test for two means):** May-2025, Oct-2023. **Pair of dice thrown 360 times (χ^2):** May-2025, Oct-2023, Nov-2020.
- **F-test for equality of variances** (pumpkins, health expenditure, methods A/B, SS = 84.4 / 102.6): almost every paper.
- **Paired t-test** (accidents before/after, husbands vs wives IQ). **Life expectancy 50 years** (single mean t).
- **Short questions:** state t / χ^2 distributions, conditions for χ^2 test, $F_{0.05}(15, 7)$ and $F_{0.95}(12, 15)$, $P(S_1^2/S_2^2 < 4.03)$.

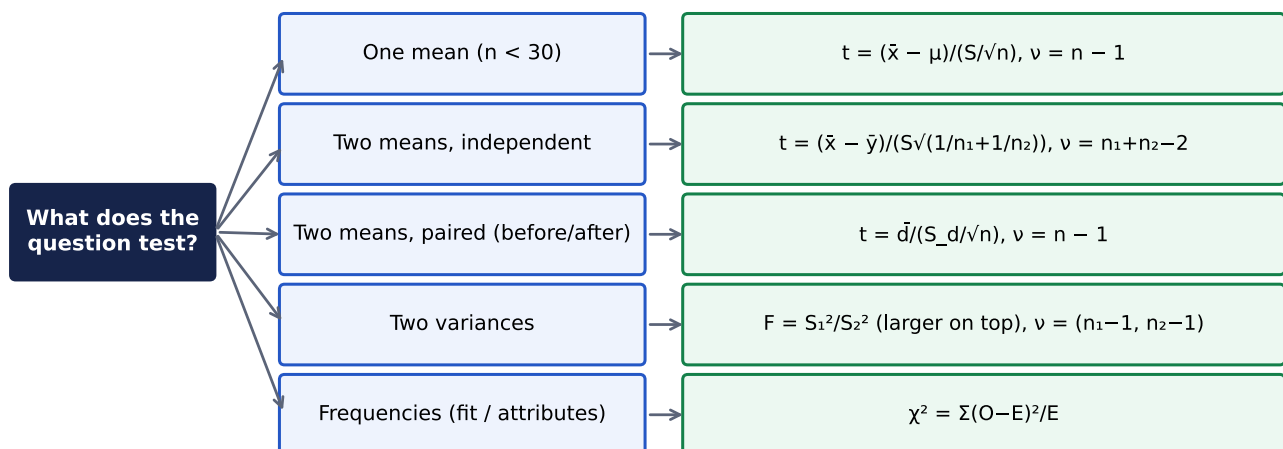


Fig 4.0 — Select the correct small-sample test from the question.

4.1 Degrees of Freedom

DEGREES OF FREEDOM (N)

The number of independent observations that make up a statistic is the **degrees of freedom**. We write it as ν .

If there are n observations and k restrictions (parameters estimated from them), then $\nu = n - k$. For one sample mean, $\nu = n - 1$.

Test	Degrees of freedom
t — single mean, paired t	$n - 1$
t — two means	$n_1 + n_2 - 2$
F — two variances	$(\nu_1, \nu_2) = (n_1 - 1, n_2 - 1)$
χ^2 — series of n classes (uniform, given ratio, binomial with known p)	$n - 1$
χ^2 — Poisson fit (λ estimated) / Normal fit (μ, σ estimated)	$n - 2$ / $n - 3$
χ^2 — contingency table $r \times c$	$(r - 1)(c - 1)$

4.2 Student's t-Distribution

DEFINITION MAY-2025 · 2M OCT-2025 · 2M

Let $x_1, x_2, \dots, x_n$ be a random sample of size n from a normal population with mean μ and variance σ^2 . The statistic

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}}, \quad \bar{x} = \frac{\sum x_i}{n}, \quad S^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

follows the **t-distribution with $\nu = n - 1$ degrees of freedom**. (S^2 is the unbiased estimate of σ^2 .) Its PDF is

$$f(t) = y_0 \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \quad -\infty < t < \infty$$

where y_0 is a constant such that $\int_{-\infty}^{\infty} f(t) dt = 1$.

NOTE — TWO EQUAL FORMS IN YOUR CLASS NOTES

If $s^2 = \frac{\sum (x - \bar{x})^2}{n}$ (divide by n), then $t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$. If $S^2 = \frac{\sum (x - \bar{x})^2}{n-1}$, then $t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$. Both give the **same t**.

Do not mix them.

PROPERTIES OF THE T-CURVE

1. Bell-shaped and symmetric about $t = 0$.
2. Mean = median = mode = 0.
3. The tails are thicker than the normal curve.
4. The shape depends only on ν .
5. As $\nu \rightarrow \infty$ ($n \geq 30$), $t \rightarrow$ standard normal.
6. Variance = $\frac{\nu}{\nu-2}$ ($\nu > 2$), more than 1.

APPLICATIONS OF T

1. Test for a single mean (small sample, σ unknown).
2. Test for the difference of two means (independent samples).
3. Paired t-test (dependent / before-after samples).
4. Confidence interval for μ : $\bar{x} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}$.
5. Test of a correlation coefficient.

t-curve: symmetric, bell-shaped, thicker tails; → normal as ν increases

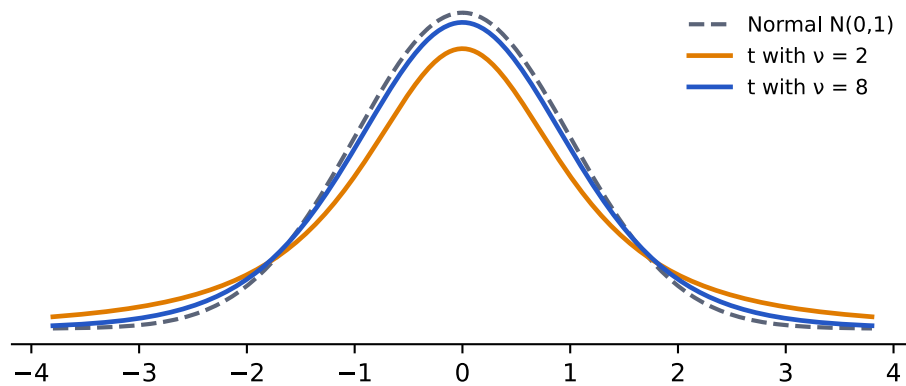


Fig 4.1 — t-curves for $\nu = 2$ and 8 against the normal curve.

4.3 t-Test for Single Mean

Step 1 $H_0: \mu = \mu_0$.

Step 2 $H_1: \mu \neq \mu_0$ (two-tailed), or $\mu > \mu_0$, or $\mu < \mu_0$.

Step 3 **LOS:** select α .

Step 4 **Test statistic:** $t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$, $S^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$, $\nu = n - 1$.

Step 5 **Tabulated value:** $t_{\alpha/2}$ (two-tailed) or t_α (one-tailed) with $\nu = n - 1$.

Step 6 **Conclusion:** If $|t| < t_{tab}$, accept H_0 . Otherwise reject H_0 .

Confidence interval: $\left(\bar{x} - t_{\alpha/2} \frac{S}{\sqrt{n}}, \bar{x} + t_{\alpha/2} \frac{S}{\sqrt{n}} \right)$.

PYQ. A random sample of 10 boys had IQs 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. (i) Do these data support the assumption of a population mean IQ of 100? (ii) Find a reasonable range in which most of the mean IQ values of samples of 10 boys lie. Nov-2020 · 8M Class notes

x	70	120	110	101	88	83	95	98	107	100	$\Sigma = 972$
$(x - \bar{x})^2$	739.84	519.84	163.84	14.44	84.64	201.64	4.84	0.64	96.04	7.84	1833.6

$$\bar{x} = 97.2, S^2 = \frac{1833.6}{9} = 203.73, S = 14.27.$$

Step 1 $H_0: \mu = 100$.

Step 2 $H_1: \mu \neq 100$.

Step 3 $\alpha = 0.05$.

Step 4 $t = \frac{97.2 - 100}{14.27/\sqrt{10}} = -0.62, |t| = 0.62, \nu = 9$.

Step 5 $t_{0.025}(9) = 2.262$.

Step 6 $0.62 < 2.262$. Accept H_0 . The mean IQ is 100.

(ii) 95% range: $97.2 \pm 2.262 \times \frac{14.27}{\sqrt{10}} = (86.99, 107.41)$.

PYQ. In 1950 in India the mean life expectancy was 50 years. A random sample of 11 persons gave life expectancies 58.2, 56.6, 54.2, 50.4, 44.2, 61.9, 57.5, 53.4, 49.7, 55.4, 57.0. Does it confirm the expected view? [Apr/May-2023 · 6M](#) [Jul-2021 · 8M](#)

$$\sum x = 598.5, \bar{x} = 54.41, \sum (x - \bar{x})^2 = 236.07, S^2 = \frac{236.07}{10} = 23.61, S = 4.859.$$

Step 1 $H_0 : \mu = 50.$

Step 2 $H_1 : \mu \neq 50.$

Step 3 $\alpha = 0.05.$

Step 4 $t = \frac{54.41 - 50}{4.859/\sqrt{11}} = \frac{4.41}{1.465} = 3.01, \nu = 10.$

Step 5 $t_{0.025}(10) = 2.228.$

Step 6 $3.01 > 2.228.$ Reject H_0 . The data do not confirm the view; the mean life expectancy is more than 50 years.

Example 1. Heights of 10 males: 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is the average height greater than 64 inches? Test at 5%. [Class notes](#)

$$\bar{x} = 66, \sum (x - \bar{x})^2 = 90, S = \sqrt{10} = 3.162. H_1 : \mu > 64 \text{ (one-tailed).}$$

$$t = \frac{66-64}{3.162/\sqrt{10}} = 2.0; t_{0.05}(9) = 1.833. 2.0 > 1.833. \text{ Reject } H_0. \text{ The average height is greater than 64 inches.}$$

Example 2. More class problems (answers). [Class notes](#)

- Orders filled in 10, 12, 19, 14, 15, 18, 11, 13 days; claim 9.5 days, $\alpha = 0.01$, $H_1: \mu > 9.5$: $\bar{x} = 14$, $S = 3.207$, $t = 3.97 > t_{0.01}(7) = 2.998$. Reject H_0 (it takes longer).
- Bulb lives (1000 h) 4.2, 4.6, 3.9, 4.1, 5.2, 3.8, 3.9, 4.3, 4.4, 5.6; test $\mu = 4000$ h: $\bar{x} = 4.4$, $S = 0.589$, $t = 2.15 < 2.262$. Accept H_0 . (Class notes get 8.51 by mixing units 4100 and 4000 with S in thousands. Keep all values in the same units.)
- Bulbs 24, 26, 30, 20, 20, 18 months, claim 25, $\alpha = 1\%$: $\bar{x} = 23$, $S = 4.52$, $|t| = 1.08 < 4.032$. Accept H_0 .
- Gutkha nicotine 2.0, 1.7, 2.1, 1.9, 2.2, 2.1, 2.0, 1.6 vs 1.83: $\bar{x} = 1.95$, $S = 0.207$, $t = 1.64 < 2.365$. Accept H_0 .
- Pesticide bags 50, 49, 52, 44, 45, 48, 46, 45, 49, 45 vs 50 kg: $\bar{x} = 47.3$, $S = 2.67$, $|t| = 3.20 > 2.262$. Reject H_0 .
- Envelopes 12.1, 11.9, 12.4, 12.3, 11.5, 11.6, 12.1, 12.4 vs 12.35 g at 1%: $\bar{x} = 12.04$, $S = 0.346$, $|t| = 2.55 < 3.499$. Accept H_0 . 95% limits: $12.04 \pm 2.365 \frac{0.346}{\sqrt{8}} = (11.75, 12.33)$.

4.4 t-Test for Difference of Means (independent samples)

TEST STATISTIC ($N_1, N_2 < 30$; **EQUAL POPULATION VARIANCES**)

$$t = \frac{\bar{x} - \bar{y}}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \quad S^2 = \frac{\sum(x_i - \bar{x})^2 + \sum(y_i - \bar{y})^2}{n_1 + n_2 - 2} = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}, \quad \nu = n_1 + n_2 - 2$$

Use the second form of S^2 when the question gives sample s.d.s s_1, s_2 (divisor n).

PYQ. Two horses A and B were tested for the time (in seconds) to run a track. A: 28, 30, 32, 33, 33, 29, 34. B: 29, 30, 30, 24, 27, 29. Test whether the two horses have the same running capacity.

May-2025 · 10M

Oct-2023 · 10M

Class notes

x	28	30	32	33	33	29	34	$\Sigma x = 219$
$(x - \bar{x})^2$	10.80	1.65	0.51	2.94	2.94	5.22	7.37	31.43
y	29	30	30	24	27	29		$\Sigma y = 169$
$(y - \bar{y})^2$	0.69	3.36	3.36	17.36	1.36	0.69		26.83

$$\bar{x} = 31.29, \bar{y} = 28.17. S^2 = \frac{31.43 + 26.83}{7 + 6 - 2} = 5.297, S = 2.301.$$

Step 1 $H_0 : \mu_1 = \mu_2.$

Step 2 $H_1 : \mu_1 \neq \mu_2.$

Step 3 $\alpha = 0.05.$

Step 4 $t = \frac{31.29 - 28.17}{2.301 \sqrt{\frac{1}{7} + \frac{1}{6}}} = \frac{3.12}{1.280} = 2.44, \nu = 11.$

Step 5 $t_{0.025}(11) = 2.201.$

Step 6 $2.44 > 2.201.$ Reject H_0 . The two horses do **not** have the same running capacity.

CORRECTION TO CLASS NOTES

The class notes reject H_0 but then write "the horses have the same capacity". When H_0 is rejected, the conclusion is the **opposite**: they do not have the same capacity.

PYQ. Two samples: size 10, mean 15, sum of squares of deviations 90; size 12, mean 14, sum of squares 108. Test whether the samples came from the same normal population.

Nov-2020 · 7M

Class notes

Oct/Nov-2021 · 15M

Same normal population means equal variances **and** equal means. So do the F-test first, then the t-test.

F-test: $S_1^2 = \frac{90}{9} = 10, S_2^2 = \frac{108}{11} = 9.818. F = \frac{10}{9.818} = 1.02; F_{0.05}(9, 11) = 2.90. 1.02 < 2.90:$ accept $\sigma_1^2 = \sigma_2^2$.

t-test: $S^2 = \frac{90+108}{20} = 9.9, S = 3.146. t = \frac{15 - 14}{3.146 \sqrt{\frac{1}{10} + \frac{1}{12}}} = 0.74; t_{0.025}(20) = 2.086.$ Accept $\mu_1 = \mu_2$.

Both are accepted. The samples come from the same normal population.

PYQ. To test the effect of a fertilizer on rice, 24 plots were used, half treated and half untreated. Untreated: mean 4.8 quintals, s.d. 0.4. Treated: mean 5.1 quintals, s.d. 0.36. Can we conclude that the fertilizer gives a significant improvement? **Apr/May-2023 · 10M**

$$n_1 = n_2 = 12. S^2 = \frac{12(0.36)^2 + 12(0.4)^2}{22} = \frac{1.5552 + 1.92}{22} = 0.158, S = 0.3974.$$

Step 1 $H_0 : \mu_1 = \mu_2.$

Step 2 $H_1 : \mu_1 > \mu_2$ (treated is better; right-tailed).

Step 3 $\alpha = 0.05.$

Step 4 $t = \frac{5.1 - 4.8}{0.3974 \sqrt{\frac{1}{12} + \frac{1}{12}}} = \frac{0.3}{0.1622} = 1.85, \nu = 22.$

Step 5 $t_{0.05}(22) = 1.717.$

Step 6 $1.85 > 1.717.$ Reject H_0 . The fertilizer gives a significant improvement.

PYQ. Gain in weight (lbs) of pigs fed on two diets. A: 25, 32, 30, 34, 24, 14, 32, 24, 30, 31, 35, 25. B: 44, 34, 22, 10, 47, 31, 40, 30, 32, 35, 18, 21, 35, 29, 22. Do the two diets differ significantly? **Nov-2020 · 7M** [Class notes](#)

$$\bar{x} = 28, \bar{y} = 30, \sum(x - \bar{x})^2 = 380, \sum(y - \bar{y})^2 = 1410. S^2 = \frac{1790}{25} = 71.6.$$

$$t = \frac{28 - 30}{\sqrt{71.6 \left(\frac{1}{12} + \frac{1}{15}\right)}} = -0.61, \nu = 25; t_{0.025}(25) = 2.060. \text{ Accept } H_0. \text{ The diets do not differ significantly.}$$

Example 3. Medicine A: 42, 39, 48, 60, 41 kg. Medicine B: 38, 42, 56, 64, 68, 69, 62 kg. Does medicine B increase the weight significantly? [Class notes](#)

$$\bar{x} = 46, \bar{y} = 57, S^2 = \frac{290+926}{10} = 121.6, S = 11.03. H_1 : \mu_1 < \mu_2.$$

$$t = \frac{46 - 57}{11.03 \sqrt{\frac{1}{5} + \frac{1}{7}}} = -1.70; |t| = 1.70 < t_{0.05}(10) = 1.812. \text{ Accept } H_0. \text{ B does not increase weight significantly.}$$

4.5 Paired t-Test (dependent samples)

WHEN AND HOW

Use it when the two samples are **paired**: the same units are measured twice (before/after), or natural pairs (husband–wife). Let $d_i = x_i - y_i$.

$$t = \frac{\bar{d}}{S_d / \sqrt{n}}, \quad \bar{d} = \frac{\sum d_i}{n}, \quad S_d^2 = \frac{\sum (d_i - \bar{d})^2}{n - 1}, \quad \nu = n - 1$$

PYQ. Average weekly losses of worker-hours due to accidents in 10 plants before and after a safety programme: Before 45, 73, 46, 124, 33, 57, 83, 34, 26, 17. After 36, 60, 44, 119, 35, 51, 77, 29, 24, 11. Is the programme effective at the 0.05 level? Nov-2024 · 10M

$d = x - y$	9	13	2	5	-2	6	6	5	2	6	$\Sigma d = 52$
$(d - \bar{d})^2$	14.44	60.84	10.24	0.04	51.84	0.64	0.64	0.04	10.24	0.64	149.6

$$\bar{d} = 5.2, S_d^2 = \frac{149.6}{9} = 16.62, S_d = 4.077.$$

Step 1 $H_0 : \mu_d = 0$ (no effect).

Step 2 $H_1 : \mu_d > 0$ (losses decrease; right-tailed).

Step 3 $\alpha = 0.05$.

Step 4 $t = \frac{5.2}{4.077/\sqrt{10}} = 4.03, \nu = 9$.

Step 5 $t_{0.05}(9) = 1.833$.

Step 6 $4.03 > 1.833$. Reject H_0 . The safety programme is effective.

PYQ. An investigator states that husbands are more intelligent than wives. IQs of 10 couples: Husbands 117, 105, 97, 105, 123, 109, 86, 78, 103, 107. Wives 106, 98, 87, 104, 116, 95, 90, 69, 108, 85. Test with a reasonable test at 0.05. Nov/Dec-2022 · 10M Class notes

Each husband is paired with his wife. So the **reasonable test is the paired t-test**.

$$d: 11, 7, 10, 1, 7, 14, -4, 9, -5, 22. \bar{d} = 7.2, \sum (d - \bar{d})^2 = 603.6, S_d = 8.19.$$

Step 1 $H_0 : \mu_d = 0$.

Step 2 $H_1 : \mu_d > 0$ (right-tailed).

Step 3 $\alpha = 0.05$.

Step 4 $t = \frac{7.2}{8.19/\sqrt{10}} = 2.78, \nu = 9$.

Step 5 $t_{0.05}(9) = 1.833$.

Step 6 $2.78 > 1.833$. Reject H_0 . Husbands are more intelligent than wives.

Class notes treat the samples as independent: $\bar{x} = 103, \bar{y} = 95.8, S = 13.51, t = 1.19 < 1.734$, accept H_0 . Write the paired test in the exam, and mention the reason (data are in pairs).

Example 4. (a) Increase in marks of 8 students after coaching: 4, -2, 6, -8, 12, 5, -7, 2. (b) Change in blood pressure of 12 patients: 5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6. Class notes

(a) $\bar{d} = 1.5, S = 6.80, t = 0.62 < t_{0.05}(7) = 1.895$. Accept H_0 : no gain from coaching.

(b) $\bar{d} = 2.58, S = 3.09, t = 2.90 > t_{0.05}(11) = 1.796$. Reject H_0 : the stimulus increases blood pressure.

4.6 F-Distribution and F-Test for Equality of Variances

F-DISTRIBUTION

Let S_1^2, S_2^2 be the unbiased variances of two independent samples of sizes n_1, n_2 from normal populations with equal variances. Then

$$F = \frac{S_1^2}{S_2^2}, \quad S_1^2 = \frac{\sum (x_i - \bar{x})^2}{n_1 - 1} = \frac{n_1 s_1^2}{n_1 - 1}, \quad S_2^2 = \frac{\sum (y_i - \bar{y})^2}{n_2 - 1} = \frac{n_2 s_2^2}{n_2 - 1}$$

follows the **F-distribution with** $(\nu_1, \nu_2) = (n_1 - 1, n_2 - 1)$ degrees of freedom.

PROPERTIES OF F OCT-2025 · 5M

1. F is always positive; the curve is in the first quadrant.
2. The curve is right-skewed (not symmetric).
3. It depends only on ν_1, ν_2 , not on population parameters.
4. The order of (ν_1, ν_2) matters: $F_{1-\alpha}(\nu_1, \nu_2) = \frac{1}{F_\alpha(\nu_2, \nu_1)}$.
5. Mean = $\frac{\nu_2}{\nu_2 - 2}$ ($\nu_2 > 2$).

APPLICATIONS OF F

1. Test for equality of two population variances.
2. To check the condition $\sigma_1 = \sigma_2$ before the two-mean t-test.
3. Analysis of variance (ANOVA).
4. Test of regression / multiple correlation.

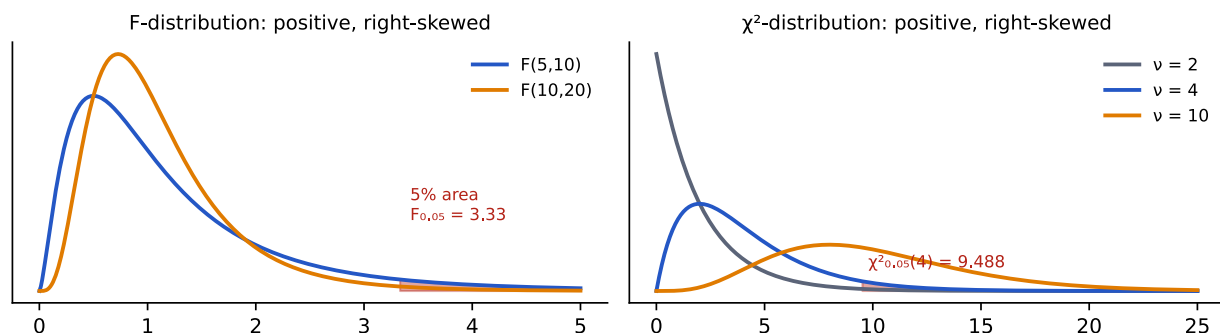


Fig 4.2 — F and χ^2 curves are positive and right-skewed. The red area is the 5% critical region.

F-test procedure Nov/Dec-2022 · 3M

- Step 1** $H_0 : \sigma_1^2 = \sigma_2^2$.
- Step 2** $H_1 : \sigma_1^2 \neq \sigma_2^2$.
- Step 3** Select α .
- Step 4** $F = \frac{\text{larger } S^2}{\text{smaller } S^2}$, with $v = (n-1 \text{ of the numerator sample, } n-1 \text{ of the denominator sample})$.
- Step 5** Read $F_\alpha(\nu_1, \nu_2)$ from the F-table.
- Step 6** If $F < F_\alpha$, accept H_0 . Otherwise reject H_0 .

WHY IS THE LARGER VARIANCE PLACED IN THE NUMERATOR? APR/MAY-2023 · 4M

The F-tables give only the **upper-tail** critical values, which are all greater than 1. When the larger variance is on top, $F \geq 1$. So we compare it directly with the table value. It also keeps one simple rule: reject H_0 only when F is too large.

PYQ. Find (i) $F_{0.05}$ for $\nu_1 = 15, \nu_2 = 7$ (ii) $F_{0.95}$ for $\nu_1 = 12, \nu_2 = 15$. May-2025 · 3M Oct-2023 · 3M

(i) From the table, $F_{0.05}(15, 7) = 3.51$. (ii) $F_{0.95}(12, 15) = \frac{1}{F_{0.05}(15, 12)} = \frac{1}{2.62} = 0.382$.

PYQ. If S_1^2 and S_2^2 are the variances of independent random samples of sizes n_1 and n_2 from normal populations with equal variances, find $P\left(\frac{S_1^2}{S_2^2} < 4.03\right)$. Nov-2024 · 3M Mar/Apr-2024 · 3M

The source problem (Miller & Freund) has $n_1 = 10, n_2 = 15$. So $F = \frac{S_1^2}{S_2^2}$ has $(\nu_1, \nu_2) = (9, 14)$.
From the F-table, $F_{0.01}(9, 14) = 4.03$. So $P(F > 4.03) = 0.01$ and $P\left(\frac{S_1^2}{S_2^2} < 4.03\right) = 1 - 0.01 = 0.99$.

0.99

In the exam, write the assumed sample sizes $n_1 = 10, n_2 = 15$ in the first line.

PYQ. Pumpkins: two samples of 11 and 9 have sample s.d.s 0.8 and 0.5. Test that the true variances are equal against the alternative that they are not, at the 10% level. Nov-2024 · 10M Class notes

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{11(0.64)}{10} = 0.704, S_2^2 = \frac{9(0.25)}{8} = 0.281.$$

Step 1 $H_0 : \sigma_1^2 = \sigma_2^2.$

Step 2 $H_1 : \sigma_1^2 \neq \sigma_2^2.$

Step 3 $\alpha = 0.10$ (two-tailed $\rightarrow$ use $F_{0.05}$ in the upper tail).

Step 4 $F = \frac{0.704}{0.281} = 2.51, \nu = (10, 8).$

Step 5 $F_{0.05}(10, 8) = 3.35.$

Step 6 $2.51 < 3.35$. Accept H_0 . The variances are equal.

PYQ. In one sample of 8 observations, the sum of squares of deviations from the mean is 84.4. In another sample of 10 it is 102.6. Test at 5% whether the populations have the same variance.

Jul-2021 · 15M Class notes

$$S_1^2 = \frac{84.4}{7} = 12.06, S_2^2 = \frac{102.6}{9} = 11.40. F = \frac{12.06}{11.40} = 1.06, \nu = (7, 9). F_{0.05}(7, 9) = 3.29. 1.06 < 3.29. \text{ Accept } H_0: \text{ same variance.}$$

PYQ. Household net expenditure on health care (% of income). South: 15, 8.0, 3.8, 6.4, 27.4, 19.0, 35.3, 13.6. North: 18.8, 23.1, 10.3, 8.0, 18.0, 10.2, 15.2, 19.0, 20.2. Test the equality of variances.

Mar/Apr-2024 · 10M Class notes

$$\text{South: } n_1 = 8, \bar{x} = 16.06, \sum(x - \bar{x})^2 = 823.18, S_1^2 = \frac{823.18}{7} = 117.60.$$

$$\text{North: } n_2 = 9, \bar{y} = 15.87, \sum(y - \bar{y})^2 = 219.50, S_2^2 = \frac{219.50}{8} = 27.44.$$

Step 1 $H_0 : \sigma_1^2 = \sigma_2^2.$

Step 2 $H_1 : \sigma_1^2 \neq \sigma_2^2.$

Step 3 $\alpha = 0.05.$

Step 4 $F = \frac{117.60}{27.44} = 4.29, \nu = (7, 8).$

Step 5 $F_{0.05}(7, 8) = 3.50.$

Step 6 $4.29 > 3.50$. Reject H_0 . The variances are not equal.

PYQ. Time taken by workers using two methods. A: 20, 16, 26, 27, 23, 22. B: 27, 33, 42, 35, 32, 34, 38. Do the variances differ significantly? **Mar/Apr-2024 · 10M**

A: $\bar{x} = 22.33$, $\sum(x - \bar{x})^2 = 81.33$, $S_1^2 = \frac{81.33}{5} = 16.27$. B: $\bar{y} = 34.43$, $\sum(y - \bar{y})^2 = 133.71$, $S_2^2 = \frac{133.71}{6} = 22.29$.

Larger on top: $F = \frac{22.29}{16.27} = 1.37$, $\nu = (6, 5)$. $F_{0.05}(6, 5) = 4.95$. Accept H_0 . The variances do not differ significantly.

PYQ. Sample A: 10, 12, 14, 16, 18. Sample B: 8, 10, 12, 14, 16. Test for equality of variances.

Oct-2025 · 5M

Both samples have the same spread: $\sum(x - \bar{x})^2 = 40$ for each, so $S_1^2 = S_2^2 = 10$. $F = 1.00 < F_{0.05}(4, 4) = 6.39$. Accept H_0 .

4.7 χ^2 -Distribution and χ^2 -Test

χ^2 -DISTRIBUTION **APR/MAY-2026 · 2M** **NOV-2024 · 2M**

Let $O_1, \dots, O_n$ be the **observed frequencies** and $E_1, \dots, E_n$ the **expected frequencies** (from a probability rule). Then

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

follows the χ^2 -distribution with ν degrees of freedom. (Also: if $Z_1, \dots, Z_\nu$ are independent $N(0, 1)$, then $\sum Z_i^2$ is χ^2 with ν d.f.) Check: $\sum O_i = \sum E_i = N$.

PROPERTIES OF χ^2 **APR/MAY-2023 · 3M**

1. $\chi^2 \geq 0$; the curve is right-skewed.
2. Mean = ν , variance = 2ν .
3. It depends only on ν .
4. As ν increases, the curve becomes nearly normal.
5. The sum of independent χ^2 variates is a χ^2 variate.

CONDITIONS FOR VALIDITY OF χ^2 TEST

MAR/APR-2024 · 2M **OCT-2023 · 2M**

1. Total frequency N is large ($N \geq 50$).
2. Each expected frequency $E_i \geq 5$. Pool small classes.
3. The observations are independent.
4. The constraints on cell frequencies are linear (e.g. $\sum O = \sum E$).

APPLICATIONS OF χ^2

(1) Goodness of fit. (2) Independence of attributes. (3) Test of a specified population variance: $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$.

χ^2 -test for goodness of fit **Oct-2025 · 3M**

Step 1 H_0 : the data fit the given distribution (the die is fair, the coins are balanced ...).

Step 2 H_1 : the data do not fit.

- Step 3** Select α .
- Step 4** Find $E_i = N \cdot p_i$. Calculate $\chi^2 = \sum \frac{(O-E)^2}{E}$.
- Step 5** Read $\chi^2_{\alpha}(\nu)$ with the correct ν .
- Step 6** If $\chi^2_{cal} < \chi^2_{tab}$, accept H_0 (good fit). Otherwise reject H_0 .

PYQ. A pair of dice is thrown 360 times. Frequencies of sums 2–12: 8, 24, 35, 37, 44, 65, 51, 42, 26, 14, 14. Are the dice fair at the 0.05 level? [May-2025 · 10M](#) [Oct-2023 · 10M](#) [Nov-2020 · 8M](#) [Class notes](#)

For fair dice, $P(\text{sum} = k) = \frac{1,2,3,4,5,6,5,4,3,2,1}{36}$. $E = 360 \cdot P$.

Sum	2	3	4	5	6	7	8	9	10	11	12
O	8	24	35	37	44	65	51	42	26	14	14
E	10	20	30	40	50	60	50	40	30	20	10
$\frac{(O-E)^2}{E}$	0.4	0.8	0.833	0.225	0.72	0.417	0.02	0.1	0.533	1.8	1.6

- Step 1** H_0 : the dice are fair.
- Step 2** H_1 : the dice are not fair.
- Step 3** $\alpha = 0.05$.
- Step 4** $\chi^2 = 7.45$, $\nu = 11 - 1 = 10$.
- Step 5** $\chi^2_{0.05}(10) = 18.307$.
- Step 6** $7.45 < 18.307$. Accept H_0 . The dice are fair.

PYQ. A die is thrown 120 times. (a) Faces 1–6: 20, 21, 19, 18, 22, 20. (b) Faces 1–6: 10, 15, 25, 25, 18, 27. Test whether the die is fair at 5%. [Oct-2025 · 5M](#) [Jul-2021 · 15M](#)

$E = 20$ for each face, $\nu = 5$, $\chi^2_{0.05}(5) = 11.07$.

(a) $\chi^2 = \frac{0+1+1+4+4+0}{20} = 0.5 < 11.07$. Accept: fair.

(b) $\chi^2 = \frac{100+25+25+25+4+49}{20} = \frac{228}{20} = 11.4 > 11.07$. Reject: the die is not fair.

PYQ. The numbers of automobile accidents per week in a community: 12, 8, 20, 2, 14, 10, 15, 6, 9, 4. Are these figures consistent with the belief that accident conditions were the same during this 10-week period? [Mar/Apr-2022 · 9M](#)

Total = 100, so $E = 10$ for each week. $\chi^2 = \frac{4+4+100+64+16+0+25+16+1+36}{10} = 26.6$, $\nu = 9$, $\chi^2_{0.05}(9) = 16.92$. $26.6 > 16.92$. Reject H_0 . The conditions were not the same.

PYQ. Fit a Poisson distribution to the data and test the goodness of fit at 0.05: x : 0, 1, 2, 3, 4; f : 419, 352, 154, 56, 19. [Nov/Dec-2022 · 10M](#)

- Step 1** $N = 1000$, $\lambda = \bar{x} = \frac{0+352+308+168+76}{1000} = 0.904$.
- Step 2** $P(0) = e^{-0.904} = 0.4049$. Use $P(r+1) = \frac{\lambda}{r+1}P(r)$: expected $E = 404.9, 366.1, 165.5, 49.9$, and the last class (4 or more) = $1000 - 986.3 = 13.7$.
- Step 3** $\chi^2 = \frac{(419-404.9)^2}{404.9} + \frac{(352-366.1)^2}{366.1} + \frac{(154-165.5)^2}{165.5} + \frac{(56-49.9)^2}{49.9} + \frac{(19-13.7)^2}{13.7} = 4.67$.

Step 4 $\nu = n - 2 = 5 - 2 = 3$ (one parameter λ estimated). $\chi^2_{0.05}(3) = 7.815$.

Step 5 $4.67 < 7.815$. Accept H_0 . The Poisson distribution is a good fit.

Example 5. 4 coins were tossed 160 times. Heads 0–4: 17, 52, 54, 31, 6. Test the goodness of fit assuming the coins are balanced. [Class notes](#)

$E = 160 \cdot \frac{1}{16} = 10, 40, 60, 40, 10$. $\chi^2 = 4.9 + 3.6 + 0.6 + 2.025 + 1.6 = 12.725$, $\nu = 4$, $\chi^2_{0.05} = 9.488$.
Reject H_0 : the coins are not balanced.

χ^2 -test for independence of attributes Apr/May-2023 · 2M Oct-2025 · 5M

CONTINGENCY TABLE METHOD

H_0 : the two attributes are independent (no association).

Expected frequency of each cell: $E = \frac{\text{Row total} \times \text{Column total}}{\text{Grand total}}$. $\chi^2 = \sum \frac{(O-E)^2}{E}$, $\nu = (r-1)(c-1)$.

For a 2×2 table with cells a, b (row 1) and c, d (row 2): $\chi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$, $\nu = 1$.

Example 6. Hair colour and eye colour of 150 persons. Rows (eye): Blue 15, 5, 20; Grey 20, 10, 20; Brown 25, 15, 20. Columns (hair): Fair, Brown, Black. Test the association. [Class notes](#)

Row totals 40, 50, 60; column totals 60, 30, 60; $N = 150$.

E : 16, 8, 16 / 20, 10, 20 / 24, 12, 24. $\chi^2 = 0.0625 + 1.125 + 1 + 0 + 0 + 0 + 0.0417 + 0.75 + 0.667 = 3.65$,
 $\nu = 4$, $\chi^2_{0.05} = 9.488$. Accept H_0 : hair and eye colours are independent.

Example 7. Soft drink preference (Pepsi, Thums Up, Fanta) of clerks, teachers, officers: Pepsi 10, 25, 65; Thums Up 15, 30, 65; Fanta 50, 60, 30. [Class notes](#)

E : 21.43, 32.86, 45.71 / 23.57, 36.14, 50.29 / 30, 46, 64. $\chi^2 = 60.23$, $\nu = 4$. $60.23 > 9.488$. Reject H_0 : the preference depends on the category of employee.

PYQ. Fish from lake A (10): 11.5, 10.8, 11.6, 9.4, 12.4, 11.4, 12.2, 11, 10.6, 10.8 ppm. Lake B (8): 11.8, 12.6, 12.2, 12.5, 11.7, 12.1, 10.4, 12.6. The method variances are known: 0.09 (A) and 0.16 (B). Can we reject (at 5%) the claim that the lakes are equally contaminated? Oct/Nov-2021 · 8M

The population variances are **known**, so use Z (not t). $\bar{x}_A = 11.17$, $\bar{x}_B = 11.99$.

$Z = \frac{11.17 - 11.99}{\sqrt{\frac{0.09}{10} + \frac{0.16}{8}}} = \frac{-0.8175}{0.1703} = -4.80$. $|Z| > 1.96$. Reject H_0 : the lakes are not equally contaminated.

4.8 Quick Revision Sheet

Test	Statistic	d.f.
Single mean	$t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$	$n - 1$
Two means	$t = \frac{\bar{x} - \bar{y}}{S\sqrt{1/n_1 + 1/n_2}}, S^2 = \frac{\Sigma(x - \bar{x})^2 + \Sigma(y - \bar{y})^2}{n_1 + n_2 - 2}$	$n_1 + n_2 - 2$
Paired	$t = \frac{\bar{d}}{S_d/\sqrt{n}}$	$n - 1$
Variances	$F = \frac{S_1^2}{S_2^2}$ (larger on top)	$(n_1 - 1, n_2 - 1)$
Goodness of fit	$\chi^2 = \Sigma \frac{(O - E)^2}{E}$	$n - 1$ (-1 per estimated parameter)
Independence	$E = \frac{RT \times CT}{N}$	$(r - 1)(c - 1)$

Key table values (5%)

$t_{0.025}$: $v=9$: 2.262, 10: 2.228, 11: 2.201, 18: 2.101, 20: 2.086

$t_{0.05}$: $v=9$: 1.833, 18: 1.734, 22: 1.717

χ^2 and F (5%)

χ^2 : $v=1$: 3.841, 3: 7.815, 4: 9.488, 5: 11.07, 9: 16.92, 10: 18.31

$F_{0.05}$: (7,9) 3.29, (9,11) 2.90, (10,8) 3.35

t-table (critical values t_α , one tail area α)

v	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

χ^2 -table

ν	$\chi^2_{0.10}$	$\chi^2_{0.05}$	$\chi^2_{0.025}$	$\chi^2_{0.01}$
1	2.706	3.841	5.024	6.635
2	4.605	5.991	7.378	9.210
3	6.251	7.815	9.348	11.345
4	7.779	9.488	11.143	13.277
5	9.236	11.070	12.833	15.086
6	10.645	12.592	14.449	16.812
7	12.017	14.067	16.013	18.475
8	13.362	15.507	17.535	20.090
9	14.684	16.919	19.023	21.666
10	15.987	18.307	20.483	23.209
11	17.275	19.675	21.920	24.725
12	18.549	21.026	23.337	26.217
13	19.812	22.362	24.736	27.688
14	21.064	23.685	26.119	29.141
15	22.307	24.996	27.488	30.578
16	23.542	26.296	28.845	32.000
17	24.769	27.587	30.191	33.409
18	25.989	28.869	31.526	34.805
19	27.204	30.144	32.852	36.191
20	28.412	31.410	34.170	37.566
21	29.615	32.671	35.479	38.932
22	30.813	33.924	36.781	40.289
23	32.007	35.172	38.076	41.638
24	33.196	36.415	39.364	42.980
25	34.382	37.652	40.646	44.314
26	35.563	38.885	41.923	45.642
27	36.741	40.113	43.195	46.963
28	37.916	41.337	44.461	48.278
29	39.087	42.557	45.722	49.588
30	40.256	43.773	46.979	50.892

F-table ($F_{0.05}$)

$v_2 \setminus v_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89

★ Assignment — Unit 4 (5 questions)

1. State the t-distribution and write its properties and applications. [5M]
2. A machine fills packets with mean weight 50 g. A sample of 10 packets gives 49, 52, 51, 48, 50, 53, 47, 52, 51, 49 g. Test at 5% whether the machine is working properly. [10M]
3. Two independent samples: A: 16, 26, 27, 23, 24, 22; B: 33, 42, 35, 32, 28, 31. Test whether the means are equal at 5%. (Do the F-test first.) [10M]
4. In a 2×2 table, 60 of 100 males and 40 of 100 females prefer brand X. Test whether brand preference is independent of gender at 5%. [5M]
5. Theory predicts the proportion of beans in four groups as 9 : 3 : 3 : 1. In 1600 beans, the counts were 882, 313, 287, 118. Do the data agree with the theory? [10M]

Answers for self-check: Q2: $\bar{x} = 50.2$, $S = 1.93$, $t = 0.33 < 2.262$, accept. Q3: $F = \frac{22.7}{15.2} = 1.49 < 5.05$; $t = -4.18$, $|t| > 2.228$, reject. Q4: $\chi^2 = 8.0 > 3.841$, not independent. Q5: E = 900, 300, 300, 100; $\chi^2 = 4.73 < 7.815$, agree.